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THE GENESIS OF POSSIBLE WORLDS SEMANTICS

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ABSTRACT. This article traces the development of possible worlds semantics through the work of: Wittgenstein, 1913–1921; Feys, 1924; McKinsey, 1945; Carnap, 1945–1947; McKinsey, Tarski and Jónsson, 1947–1952; von Wright, 1951; Becker, 1952; Prior, 1953–1954; Montague, 1955; Meredith and Prior, 1956; Geach, 1960; Smiley, 1955–1957; Kanger, 1957; Hintikka, 1957; Guillaume, 1958; Binkley, 1958; Bayart, 1958–1959; Drake, 1959–1961; Kripke, 1958–1965.

KEY WORDS: history of logic, modal logic, possible worlds semantics

Of the past only that part is real today which is still active today in its effects.
Jan Łukasiewicz

1. Introduction

I shall say nothing here concerning the early history of possible worlds semantics and the work of such figures as Duns Scotus, William of Ockham, John Wallis, and Leibniz. The topic of enquiry is the modern or technical era of possible worlds semantics. This era perhaps began with the work of Peirce, who advocated an analysis of the conditional in terms of quantification over possible worlds:

> The quantified subject of a hypothetical proposition is a *possibility*, or *possible case*, or *possible state of things*. (Hartshorne and Weiss, 1932, 2.347)

> an ordinary Philonian conditional [if A then B] is expressed by saying, 'In any possible state of things, i, either [A] is not true [in i] or [B] is true [in i]'. (Hartshorne and Weiss, 1933, 3.444; see also 3.374)

There are three principal threads to the history of possible worlds semantics. First, there is the idea that the modalities are to be analyzed in terms of quantifications over possibilia. Second, there is the use of the binary relation (or equivalent). Often described as being a relation of accessibility between worlds, this is the key to obtaining semantics for systems weaker than the Lewis system S5. For example, if it is required that the binary relation be reflexive then the modal formula $\Box A \rightarrow A$ – the characteristic axiom of a system formulated independently by Gödel, Feys, and von Wright, known variously as M and T – is validated in

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the semantics. If the binary relation is transitive, the S4 reduction axiom $\Box A \rightarrow \Box\Box A$ is validated; if symmetrical, the Brouwershe (or B) axiom $A \rightarrow \Box\Diamond A$ is validated; and if transitive and symmetrical, the S5 reduction axiom $\Diamond A \rightarrow \Box\Diamond A$ is validated.

As early as 1946, Carnap explored the idea of analyzing modality as quantification over possible worlds, but did not have the binary relation. During the 1940s and early 1950s, various logicians introduced the binary relation, or an equivalent, but not in connection with possible worlds. It seems that Meredith and Prior were the first to give a possible worlds semantics employing the binary relation, in joint work in 1956. Meredith and Prior were influenced both by the passages from Peirce just quoted and by the account of modality given by Wittgenstein in the *Tractatus*. They in effect gave soundness theorems for the systems M, S4, and S5, showing that the axioms of S5 are valid if the binary relation between worlds – for which Prior subsequently coined the term 'accessibility relation' – is reflexive, transitive and symmetrical; and mutatis mutandis in the case of S4 and M.

The third thread of the story is the quest for completeness proofs. In the case of propositional modal systems, the story is complex. The first person to have announced completeness proofs for propositional M, S4 and S5 relative to a semantics explicitly interpreted in terms of the notion of a possible world appears to have been Smiley in 1957. The earliest completeness proofs for quantified systems weaker than S5 appear to have been obtained by Hintikka and Kripke, in a glorious photo-finish.

I shall present the story in roughly chronological order, beginning with Wittgenstein, whose remarks in the *Tractatus* on logically necessary and logically impossible propositions were a strong influence not only on Meredith and Prior but also on Carnap.

2. Wittgenstein, 1913–1921

In Wittgenstein's 'Notes on Logic' (1913) are to be found the glimmerings of the matrix (or value-table) account of possible worlds.[1]

> the two poles [are] *true* and *false*. ... Let *n* propositions be given. I then call a 'class of poles' of these propositions every class of *n* members, of which each is a pole of one of the *n* propositions, so that one member corresponds to each proposition. (Third MS; Wittgenstein, 1979: 102)

In the Fourth MS, Wittgenstein said:

> If we formed all atomic propositions, the world would be completely described if we declared the truth or falsehood of each. (1979: 103)

There is no explicit statement to the effect that the remaining vectors in the class of poles describe possible states-of-affairs, and nor is there any explicit attempt, in 'Notes on Logic', to link this apparatus to the modal notions. However, in a letter to Russell written in November 1913, Wittgenstein gave a procedure in terms of poles for distinguishing propositions of the propositional calculus into the 'true logical', the 'false and logical', and those that are not logical at all (1979: 125-6). Later statements from the 'Notebooks' connect the notions of 'tautology' and 'contradiction' with the range of possibilities:

Why does tautology say nothing? Because every possibility is admitted in it in advance . . . (5 June 1915; 1979: 55)

One might simply say: 'p.∼p' says nothing in the proper sense of the word. For in advance there is no possibility left which it can *correctly* present. (13 June 1915; 1979: 59)

By the time of the *Tractatus*, Wittgenstein had drawn these ideas together into an informal account of the truth-conditions of tautologies and contradictions in terms of truth (falsity) in all possible combinations of states-of-affairs:

If an elementary proposition is true, the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist. If all true elementary propositions are given, the result is a complete description of the world. . . . For n states of affairs, there are $K_n = \sum_{v=0}^{n} \binom{n}{v}$ possibilities of existence and non-existence. Of these states of affairs any combination can exist and the remainder not exist. There correspond to these combinations the same number of possibilities of truth – and falsity – for n elementary propositions. Truth-possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs. . . . Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions. . . . Among the possible groups of truth-conditions there are two extreme cases. In one of these cases the proposition is true for all the truth-possibilities of the elementary propositions. . . . In the second case the proposition is false for all the truth-possibilities . . . In the first case we call the proposition a tautology; in the second, a contradiction. (*Tractatus*, 4.25-4.3, 4.41, 4.46)

In Carnap's hands, these ideas were to blossom into a formal semantics for S5 (Carnap, 1946, 1947); see Section 5.

3. Feys, 1924

In an informal and little-known essay written in French, Feys gave an analysis of the four Aristotelian modalities in terms of possible cases ('des cas possibles'):

A given conception of the truth is defined by the gathering, the grouping of all judgements that one supposes to be true. . . . One can imagine as many 'possible conceptions' of the truth as one can have reconcilable (compossible) combinations of all the conceivable judgements. (1924: 318)[2]

[T]he totality Ω is the totality of all possible conceptions of the truth, of all the cases where some propositions are true; the null class 0 is the class empty of all possibility of truth, the absurd. . . .

1) $P = 0$. The proposition P is not true in any case. P is absurd, *impossible*.
2) $P \neq 0$. The class of cases where P is true is not null. The proposition P is *possible*.
3) $P = \Omega$. P is true in the totality of possible cases. The proposition P is *necessary*.
4) $P \neq \Omega$. The class of cases where P is true does not coincide with the totality of possible cases. P is *contingent*.

Here we have precisely the four types of *modal proposition* of Aristotelian logic. (1924: 320–321)

In a posthumously published monograph (written in English) Feys in effect reiterated this account (1965: 154). He introduced the concept of a 'case-abstract' $\hat{t}$, saying:

The abstract '$\hat{t}$ pt' . . . expresses the property of being a case t such that pt, or the class of cases where p is true; '$\hat{t}$ pt' is an event taken abstractly, what is common between the realizations of an event p in different cases. (1965: 153–154)

When describing how expressions containing t-variables are to be translated into expressions of the ordinary modal calculus, Feys identified the universal quantification '$\forall t$' with '$\Box$' and the existential '$\exists t$' with '$\Diamond$' (1965: 160–161). However, in discussing 'the semantical interpretation of this translation', Feys distanced himself somewhat from the idea of possible worlds, alluding to 'difficulties raised by Quine, Prior, and others' (1965: 160), saying:

we are not bound to an ontological interpretation of the [individual variables] x, y, z . . . as permanent realities through possible worlds. (ibid.)

Becker (1952) was to utilise these ideas from Feys (1924) in formulating his 'statistical interpretation' of the modalities (Section 8).

4. McKinsey, 1945

In a paper entitled 'On the Syntactical Construction of Systems of Modal Logic', McKinsey presented what he called a 'syntactical definition' of possibility (remarking that his definition 'is based on certain ideas of Carnap [1937: 181, 250–255]' (1945: 83)):

As the intuitive basis for the syntactical definition of possiblity, I take the position that to say a sentence is possible means that there exists a true sentence of the same form. Thus, for example, it would be said that the sentence, 'Lions are indigenous to Alaska,' is possible, because of the fact that the sentence, 'Lions are indigenous to Africa,' has the same form and is true. But unfortunately the question, when two sentences have the same form, is not easily answered without becoming involved in philosophical questions. ... I shall avoid these difficulties, by supposing merely that we are given a certain set of substitutions which take the sentences of a certain language into other (or sometimes the same) sentences of the language; and I shall call a sentence possible, if some substitution of the set takes it into a true sentence. (1945: 83)

McKinsey gave various conditions on a set S of substitutions and obtained soundness results for S4 and S5. In the case of S4, the crucial condition is:

If s_m and s_n are any two elements of S, then there is an element s_t of S such that, for every sentence α of L, $s_t(\alpha) = s_n(s_m(\alpha))$. (ibid.: 84)

This closure condition is the functional equivalent of the relational condition of transitivity. If one is thinking in terms of functions rather than relations – and a formal semantics may be expressed equivalently in terms of either – then the postulate of reflexivity becomes the postulate of an identity function (in McKinsey's terms, the existence of a substitution s_1 such that for every α in L, $s_1(\alpha) = \alpha$) and the postulate of symmetry becomes the postulate of an inverse function (in McKinsey's case, the existence of, for each substitution s_m and every finite set F of formulae of L, a substitution s_n such that $s_n(s_m(\alpha)) = \alpha$ for each α in L). Since McKinsey's approach to modality was based on the idea of mapping formulae onto isomorphic formulae, it was natural that he should formulate the conditions required for his soundness results in functional rather than relational terms.

McKinsey's soundness results were in terms of a set T of true sentences, which he defined recursively, using the following condition for $\Diamond$:

$$\Diamond\alpha \text{ is in } T \text{ if and only if there exists an element } s_n \text{ of } S \text{ such that } s_n(\alpha) \text{ is in } T.$$

McKinsey proved that if S satisfies the identity and closure postulates, then every theorem of S4 is in T, and that if S in addition satisfies the inverse postulate, then every theorem of S5 is in T. These appear to be the earliest soundness theorems for propositional S4 and S5.[3]

Although McKinsey's approach makes no use of the notion of a possible world, or even of a model, his account does have a semantical flavour. Smiley dubs the approach 'translational semantics', saying

McKinsey's word 'syntactical' is a terrible misnomer, since what he is presenting is straightforwardly a semantics. Etchemendy [1990] distinguishes 'interpretational' from 'representational' semantics, and what McKinsey was doing ... is precisely ... interpretational semantics.[4]

5. Carnap, 1945–1947

In his paper 'Modalities and Quantification' (1946) and his volume *Meaning and Necessity* (1947), Carnap turned his back on the syntactical account of modality that he offered in his *The Logical Syntax of Language* (1937). He treated the modality 'L-truth', which, he said, 'is meant as an explicatum for what Leibniz called necessary truth and Kant analytic truth' (1947: 8). Carnap emphasized that 'the definition of L-truth here chosen ... is based on Wittgenstein's conception of the nature of logical truth' (1946: 47). Carnap attempted a possible worlds semantics for quantified S5, based on his idea of a *state-description*. His 1946 paper, which contained the first technical work in possible worlds semantics, is remarkable in its scope. The paper offers the thesis of the necessity of identity, semantical validations of the Barcan formula and its converse, and much else besides.

A state-description is

> a class of sentences which represents a possible specific state of affairs by giving a complete description of the universe of individuals with respect to all properties and relations designated by predicates in the system. ... [A] state-description ... contains for every atomic sentence S_i either S_i itself or $\sim S_i$, but not both. ... That [a sentence] holds in a given state-description means, in non-technical terms, that this state-description entails [the sentence]; in other words, that [the sentence] would be true if this state-description were the description of the actual state of the universe. (1946: 50)

Carnap remarked that

> the state-descriptions represent Leibniz' possible worlds or Wittgenstein's possible states of affairs (1947: 9)

and that

> Leibniz' conception [was] that a necessary truth must hold in all possible worlds [and] [s]ince our state-descriptions represent the possible worlds, this means that a sentence is logically true if it holds in all state-descriptions. (1947: 10)

Carnap's approach was different from that followed in modern possible worlds semantics, in that he evaluated modal sentences over a single class, the class of all state-descriptions, which he referred to as the *universal range*. A statement is said to be L-true if and only if it holds in every state-description in the universal range, and to be L-false ('impossible') if and only if it holds nowhere in the universal range (1946: 50–51, 54). In his 1946 paper Carnap announced soundness proofs for propositional S5 and quantified S5, but he left the question of completeness open (1946, Sect. 12). It was subsequently discovered – it seems by Montague and Kalish[5] – that Carnap's semantics is incomplete. The set of all L-true sentences (of the language in question) is not recursively enumerable and therefore is not axiomatisable (Cocchiarella, 1975). Success in establishing a completeness theorem for quantified S5 was eventually achieved by setting aside Carnap's natural approach, whereby $\Box A$ is evaluated in terms of a quantification over *all* possible worlds (relative to a given universe of individuals and given class of predicates), replacing it instead with what Cocchiarella has justly described as a 'model-theoretic artifice'. In the later semantics for quantified S5, $\Box A$ is said to be true in a model if and only if A is true in every possible world countenanced by that model, where these worlds may form a proper subset of the set of all worlds countenanced by the model theory *in toto*.

6. McKinsey, Tarski and Jónsson, 1947–1952

In 1948, McKinsey and Tarski gave algebraic characterisations of the systems S4 and S5 (McKinsey and Tarski, 1948).[6] Corresponding to the S4 axiom was the condition that the algebra be a closure algebra, and to the S5 axiom the condition that every closed element also be open (and so every open element closed).[7] Their idea of enriching Boolean algebras with new operations was extended to arbitrary Boolean algebras with operators by Jónsson and Tarski (1948, 1951, 1952). Jónsson and Tarski proved rather general theorems connecting Boolean algebras with sets having certain relations between their elements. Their theorem 3.5 (1951: 930–93l) states equivalences between various functional and relational conditions, the latter including reflexivity, transitivity and symmetry; and in theorem 3.14 (1951: 935) they showed that every closure algebra is isomorphic to an algebraic system formed by a set and a reflexive and transitive relation between its elements.

With hindsight, these theorems can be viewed as in effect a treatment of all the basic modal axioms and corresponding properties of the accessibility relation. Kripke described this paper by Jónsson and Tarski as the 'most surprising anticipation' of his own work (1963a: 69). Later Kripke expanded on this remark:

> Had they [Jónsson and Tarski] known that they were doing modal logic, they would have had the completeness problem for many of the modal propositional systems wrapped up, and some powerful theorems. Mathematically they did this, but it was presented as algebra with no mention of semantics, modal logic, or possible worlds, let alone quantifiers. When I presented my paper at the conference in Finland in 1962, I emphasised the importance of this paper. Tarski was present, and said that he was unable to see any connection with what I was doing![8]

In the light of Tarski's previous work with McKinsey, this remark is perhaps puzzling.

7. VON WRIGHT, 1951

In his *Essay on Modal Logic* von Wright wrote:

One should, however, not fail to observe that there are essential similarities between alethic, epistemic, and deontic modalities on the one hand and quantifiers on the other hand. ... The logic of the words 'possible', 'impossible', and 'necessary', in other words, is very much similar to the logic of the words 'some', 'no', and 'all'. It is indeed not surprising that this should be the case. For, popularly speaking, the possible is that which is true under some circumstances, the impossible that which is true under no circumstances, and the necessary that which is true under all circumstances. (1951: 2, 19)

The formal analogy between the quantifiers and modal concepts suggested to von Wright that 'the use of truth-tables and normal forms as decision methods in quantification theory ... might, with due modifications, be transferred to modal logic' (1951: v). Von Wright expounded a truth-table method for modal formulae, the various subsidiary columns of the table exhibiting the values of the disjuncts of the target formula's disjunctive normal form. As in Wittgenstein's system, combinations of truth values correspond to possibilities. Von Wright explained that the (absolutely perfect) disjunctive normal form

shows with which ones of a finite number of mutually exclusive and jointly exhaustive possibilities the ... sentence in question expresses agreement and with which ones it expresses disagreement. If it agrees with all possibilities it expresses a truth of logic. (1951: 24–25)

Von Wright proved the completeness of his system M″, equivalent to S5 (ibid.: 85–87). His method for proving completeness makes essential use of the S5 reductions $\Box\Diamond A = \Diamond A$ and $\Box\Box A = \Box A$ (ibid.: 87).

In a later exposition, von Wright (1963) explicitly linked his normal forms with state-descriptions and possible worlds:

If there are n logically independent propositions there are evidently 2^n possible ways in which they can be true and/or false together. Any such distribution of truth-values over the n propositions will be called a *truth-combination*. ... Given n atomic formulae, one can form 2^n different conjunction-formulae such that every one of the atomic formulae or its negation-formula is a constituent in the conjunction. ... It is easily understood in which sense these 2^n different conjunction-formulae may be said to 'correspond' to the 2^n different truth-combinations in the propositions expressed by the atomic formulae. The conjunction-formulae are sometimes called *state-descriptions*. The conjunctions themselves can be called *possible worlds* (in the 'field' or 'space' of the propositions expressed by the atomic formulae). The (perfect) disjunctive normal form of a formula is a disjunction of (none or) some or all of the state-descriptions formed of its atomic constituents. If it is the disjunction of them all the formula expresses the tautology of the propositions expressed by its atomic constituents. This illustrates a sense in which a tautology can be said to be *true in all possible worlds*. If again the disjunctive normal form is 0-termed the formula expresses the contradiction of the propositions expressed by its atomic constituents. A contradiction is *true in no possible world*. Propositions which are true in some possible world(s) but not in all are called *contingent*. (1963: 19, 21–22)

8. BECKER, 1952

In his book *Untersuchungen über den Modalkalkül* (1952) [*Investigations into Modal Calculus*], Becker proposed a Leibnizian semantics for the modal operators. He called this the 'statistical interpretation of the modal calculus'.

Leibniz has already supplied a statistical theory of modalities, in his theory of possible worlds in the understanding of God, of which only one is put into being by God through his free will. The necessary truths are true in all possible worlds, the necessary falsehoods (impossibilities) are not true in any possible world. What is possible is found in at least one world, and what is unnecessary is not found in all possible worlds, i.e. not found in at least one.[9] (1952: 18)

At the start of his account Becker referrred to Feys (1924) (1952: 16). (Elsewhere in his book, Becker referred to Carnap's *Logische Syntax der Sprache* (1952: 37), but was seemingly unaware of Carnap's later work in English.)

Formally, the statistical interpretation is arrived at by means of the following definition, $P(x)$ signifying that p is true in case x:

1. $Np = (x)P(x) \equiv \sim (Ex)\sim P(x)$ Def.
2. $\sim Mp = \sim (Ex)P(x) \equiv (x)\sim P(x)$ Def.
3. $Mp = (Ex)P(x) \equiv \sim (x)\sim P(x)$ Def.
4. $\sim Np = \sim (x)P(x) \equiv (Ex)\sim P(x)$ Def.

To every modal expression thus corresponds an expression of the predicate calculus. (1952: 16)

Becker continued:

The crucial point lies in the question: How do I express p in contrast to Np? Np is said to mean 'p is true' in all cases. But what does p itself mean? We are able to come to an interpretation if we consider Leibniz's metaphysical formulation. Leibniz posits: the necessary truths (vérités nécessaires) are true in all possible worlds; only a world chosen by God's free will and thus 'realised' is, however, actual.

Translated back into the abstract formulation of logic, this means that 'actually true' signifies: is true in the designated case t. t is a definite constant or 'term', not a free variable. (1952: 17)

For example, $\Box p \to p$ is interpreted $(x)P(x) \to P(t)$ (ibid.: 17).

In order to deal with iterated modalities, Becker stratified the conceivable worlds or 'cases' (Fällen) into what he called 'case classes' (Fallklassen) (ibid.: 20). For example:

> MNp: Among the conceivable (first level) case classes there is at least one such that p is fulfilled in all individual cases contained in it.
> NMp: In all (first level) case classes there is at least one case in which p is true.
> NNp: p is true in all individual cases in all first level case classes.
> MMp: p is true in at least one case in at least one first level case class.
> $\sim M \sim Mp \equiv NMp$: In no case class is there no case in which p is true, i.e. ... in every case class there exists at least one case in which p is true.
> Higher degree forms (in which n modal symbols are graduated above one another) are represented through corresponding higher level case classes (up to and including the $n-1$th level). (ibid.: 20)

Becker shows that various of the Lewis axioms are 'proved true'. For example, $\Box(p \,\&\, q) \equiv \Box p \,\&\, \Box q$:

> If p is true in all cases, and q is true in all cases, then 'p and q' is also true in all cases, (ibid.: 22)

Both $\Box p \to \Box\Box p$ and $\Diamond p \to \Box\Diamond p$ are 'invalid in the statistical sense' (ibid.: 23–24). $\Box\Box p$ makes a stronger demand than the antecedent $\Box p$, namely that p be true in every case in every case class, whereas $\Box p$ requires only that p be true in every case in the 'designated' or 'genuine' [wirklich] case class (ibid.: 23); similarly, $\Box\Diamond p$ requires that every case class contain a case in which p is true, and the antecedent $\Diamond p$ only that the designated case class contain a case in which p is true. Becker maintained that his interpretation of the modal operators enables an 'objectively founded choice' among the systems S1–S5, reducing the 'formal possibilities ... to just a few'(ibid.: 19–20).

Neither the binary relation nor the idea of proving completeness was present in Becker's work.

9. Prior, 1953–1954

Prior appears to have been the first to use a binary relation in an explicitly modal context – in fact, a bimodal context – and the first to employ an accessibility-like interpretation of the relation. In the course of re-expressing propositions of his tense-modal logics in the form of quantifications over times, Prior introduced a relation holding between an earlier and a later point of time. Upon this relation he imposed various conditions, including for instance the condition of transitivity necessary to secure the tense-modal analogue of the S4 reduction axiom.

In his first foray into temporal logic, Prior employed Łukasiewicz's 3-valued logic, whose third value Łukasiewicz attached to propositions referring to future contingencies (Łukasiewicz, 1920, 1930; Prior, 1953). Prior soon became dissatisfied with this approach. He had the idea (with which he excitedly woke his wife, Mary, late one night in 1953) that off-the-shelf modal syntax can be used to give a representation of tense. Prior replaced the single possibility operator of standard modal logic with a pair of analogous operators, P and F, the past tense operator and the future tense operator, respectively. He then defined a pair of operators G and H analogous to the necessity operator. The definitions parallel the usual definition of the necessity operator in terms of the possibility operator: GA is $-F-A$ and HA is $-P-A$. In 1954, in the presidential address to a conference held in Wellington, New Zealand, Prior related his two primitive tense-modal operators P and F by means of a pair of what would now be called interactive axioms. The first of these axioms was $A \rightarrow GPA$ and the second $A \rightarrow HFA$. Examples of other axioms that Prior used are $FFA \rightarrow FA$ and $GA \rightarrow FA$. (The text of the address was not published until 1958, when it appeared as 'The Syntax of Time-Distinctions' (Prior, 1958).)

Prior explained in his Wellington address how his tense-logical axioms can be derived in the first-order predicate calculus enriched by axioms expressing conditions on a binary relation. In this first presentation he used the letter 'l' for the relation, reading it 'later than', but subsequently, with a view to emphasizing the greater generality of his method, he switched to the letter 'U', referring to the result of adding axioms for U to the predicate calculus as a 'U-calculus'. In the original U-calculus, or l-calculus, of the Wellington address, tensed propositions are treated as predicates expressing properties of times. Where 'p' is any proposition, tensed or not, and 'x' is a variable referring to a time, the concatenation 'px' says 'p at x'. 'Fp' is translated by the U-calculus expression $\exists x(px \,\&\, \mathrm{U}xz)$ 'z' being a free variable used to represent the time of utterance. Prior found that an axiom expressing the transitivity of U is required to prove the translation of his axiom $FFA \rightarrow FA$; and that an axiom $\forall x \exists y \mathrm{U}yx$ is required for his axiom $GA \rightarrow FA$. His other tense-modal axioms required other conditions on U, except for the two interactive axioms, which – like the alethic axiom $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$ – are available from the ordinary axioms of predicate logic.

Perhaps it was clear to Prior at the time of the Wellington address that, in principle, a parallel approach might be applied to alethic modal logic. Certainly, the idea that the modalities may be analysed in terms of quantification over *possible states of affairs* is present in a book manuscript, never published, which Prior completed in 1951, entitled ‘The Craft of Formal Logic’.[10] Prior wrote in ‘The Craft’:

> For the similarity in behaviour between signs of modality and signs of quantity, various explanations may be offered. It may be, for example, that signs of modality are just ordinary quantifiers operating upon a peculiar subject-matter, namely possible states of affairs. ... It would not be quite accurate to describe theories of this sort as ‘reducing modality to quantity’. They do reduce modal *distinctions* to distinctions of quantity, but the variables to which the quantifiers are attached retain something modal in their signification – they signify ‘possibilities’, ‘chances’, ‘possible states of affairs’, 'possible combinations of truth-values’, or the like. (pp. 736–7)

Important influences on Prior here were the *Tractatus* account of tautology and contradiction, and the writings of C. S. Peirce (in his unpublished manuscript ‘Computations and Speculations’, Prior referred to the passages from Peirce that were quoted above); Prior also mentioned John Wallis – a professor of geometry at Oxford in the 17th century – in this connection.

10. MONTAGUE, 1955

At UCLA in May 1955, Montague gave a talk entitled ‘Logical Necessity, Physical Necessity, Ethics, and Quantifiers’. Montague’s handwritten notes for the talk survive.[11] This manuscript, henceforth LNPNEQ, overlaps yet is not identical with its published successor, Montague (1960). Among the differences are the presence in LNPNEQ of two short sections not appearing in the 1960 paper, entitled ‘Ethics’ and ‘Quantifiers’, and the inclusion in the 1960 paper of a section entitled ‘A Missing Law’ not appearing in LNPNEQ.

LNPNEQ begins:

> My purpose is to give interpretations of the phrases ‘it is logically necessary that’, ‘it is physically necessary that’, and ‘it is obligatory that’. The interpretations were suggested by certain logical analogies between these phrases and the universal quantifier. The interpretations are to be given in an extensional metalanguage; in particular, the metalanguage is not to contain any of the phrases themselves. Further, the interpretation is to be such as to permit the use of the phrases in conjunction with quantifiers.

Montague treated not S5 but a nameless monomodal system of his own devising, here called S5–M. S5–M becomes equivalent to (propositional) S5 upon the addition of the M-principle $\Box A \rightarrow A$. Montague explained in ‘A Missing Law’ (1960: 268) why he omitted the M-principle: $\Box A \rightarrow A$ fails for both ethical obligation and physical necessity (in each case, some model satisfies $\Box A$ without satisfying A).

Montague extended a Tarskian definition of satisfaction-in-a-model to the modal case. He defined a model as an ordered triple $\langle D, R, f \rangle$, where

D is a domain, R is a function that assigns an appropriate extension (from D) to each predicate and individual constant, and f is a function that assigns to each individual variable a member of D. The treatment that he offered of logical necessity is this:

it seems reasonable to consider ⌜it is logically necessary that ϕ⌝ as asserting that ϕ holds under every assignment of extensions to its descriptive constants [predicate and individual constants]. (LNPNEQ: 24)

Montague borrowed the satisfaction clauses for atomic statements and truth-functional compounds 'without alteration' from Tarski. He noted that Tarski's satisfaction clause for '$\forall x$' may be thought of as involving a binary relation Q between models. $\langle D, R, f\rangle Q \langle D', R', f'\rangle$ if and only if $D = D'$, $R = R'$, and $f'(\alpha) = f(\alpha)$ for every individual variable α different from x. Reading '$\Box$' as 'for all x', the satisfaction clause for the universal quantifier becomes:

$\langle D, R, f\rangle$ satisfies $\Box\phi$ if and only if, for every model M such that $\langle D, R, f\rangle Q$ M, M satisfies ϕ.

Montague generalised this idea, allowing arbitrary binary relations between models. Where X is any such relation, *Satisfaction*$_X$ is defined by replacing 'satisfies' by '*satisfies*$_X$' in the clauses for the truth-functions and by adding the following clause for the operator $\Box$:

$\langle D, R, f\rangle$ satisfies$_X$ $\Box\phi$ iff for every model M such that $\langle D, R, f\rangle X$ M, M satisfies$_X$ ϕ.

Montague then stated a soundness theorem. Where ϕ is any formula derivable in S5–M, every model satisfies$_X$ ϕ, provided only that X fulfils the following conditions:

(i) for all M, there is an N such that MXN,
(ii) for all M, N, P, if MXN and NXP, then MXP, and
(iii) for all M, N, P, if MXN and MXP, then NXP.

Did Montague have a completeness theorem in 1955? In the 1960 paper he remarked that the deductive system is complete (1960: 264) and in a footnote compared his Tarski-style approach with Kripke's (1959a) approach (1960, note 5). Montague's editing left it unclear whether the remark in the text predated the footnote or whether both were added at the same time. Charles Silver – a student of Cocchiarella – recalls the following from 1969 or 1970:

After praising Kripke's work one time, Richard Montague mentioned that he too had a modal system as early as 1955, which was similar to Kripke's. After saying this, he paused, looked down at the table sadly and said softly, 'but no completeness proofs'.[12]

Examination of the 1955 manuscript shows that Montague did in fact assert (although does not exhibit) a completeness result for the propositional system S5–M:

a formula is valid if and only if it is a theorem. Furthermore, there is a decision method for the class of valid formulas . . . (LNPNEQ: 33)

A later version of the manuscript, partly typewritten, contains the amplification:

A proof of the completeness and decidibility of the system [S5–M] can be obtained without much difficulty from the ideas in the article of Wajsberg cited above.

This article, Wajsberg (1933), also influenced Meredith, and is discussed in Section 11.

What Montague did *not* claim is a completeness result for *quantified* S5–M. In the section of LNPNEQ entitled 'Quantifiers' (omitted from the 1960 paper), he remarked:

It has been seen that N [$\Box$] can be eliminated in favour of quantifiers in the second-order predicate calculus. . . . In fact, the theory which contains quantifiers and N (and no other modal operators) seems to lie between the first-order and the second-order predicate calculus in power of expression. The first-order calculus can be completely axiomatised; the second-order calculus cannot. There is hope that the theory with N can be completely axiomatised. (LNPNEQ: 62-64)

The previously mentioned footnote 5 of the 1960 paper sheds additional light. There Montague said that his completeness result is equivalent to the following:

A formula ϕ (of the language *S*) is a theorem of [S5–M] if and only if ϕ is satisfied by every complete model. (1960: 269)

He made it clear that language *S* contains 'no quantifiers', only individual variables and constants, predicates, truth-functional connectives, and a single operator ('N' in Montague's notation) (1960: 260; LNPNEQ: 9). Later in LNPNEQ Montague did extend the language in various ways, adding quantifiers and identity, and allowing more than one primitive modal operator; but there were no completeness claims.

Despite his use of a binary relation Montague mentioned no completeness results for systems other than S5–M. The binary relation functions only to ensure that $\Box A \rightarrow A$ is not always satisfied. Montague offered no interpretation of the binary relation, either in LNPNEQ or the 1960 paper. Montague's binary relation holds between models and not – as in Meredith and Prior (1956), Bayart (1958) and Kripke (1963a) – between points or indices interpretable as worlds (or times, in the case of Prior's 1954 work) and themselves able to form components of models. Once models are enriched with worlds (or times), each world (time) having its own set of related worlds (times), the model theory is able to countenance worlds (times) that assign the same truth values to all atomic formulae and yet are distinct. As Montague said in later work:

> The idea of using accessibility relations in connection with modal logic was introduced independently in the 1957 publication Kanger [1957a]. . ., the 1955 talk reported in Montague [1960], and the 1960 talk reported in Hintikka [1961]. In these occurrences, however, accessibility relations were always relations between models; accessibility relations between points of reference . . . appear to have been first explicitly introduced in Kripke [1963a]. (1974: 109)

(Of course, the attribution of priority to Kripke is in error, as is any implication that, prior to 1955, no one had employed the binary relation in a modal context.)

In the years that followed, Montague worked on the completeness problem for 'sentential modal logics' in association with Kalish.[13] The two obtained 'many partial results'. These results were to be presented at an APA meeting in late December 1959. Shortly before the meeting, Montague and Kalish saw an abstract in the December issue of the *Journal of Symbolic Logic* (Kripke, 1959b), announcing completeness results for a wide range of modal systems (including M, S2, S3, S4, S5, S6, S7, S8, E2, E3, E4, E5′, related systems intermediate between M and S2, systems using the Brouwersche axiom, and various systems of deontic logic). Astonished, Montague and Kalish simply withdrew their paper.[14] Kaplan recalls that Montague was curious to know whether S. Kripke was a man or a woman; everyone was surprised when Kripke turned out to be a child.[15]

In a famous review of Kripke (1963a), Kaplan stated that in 1955 'Montague suggested the interpretation of modal calculi in terms of a relation between worlds' (1966: 122). However, this is misleading, since the binary relation considered in LNPNEQ is a relation between models, and at no point in LNPNEQ did Montague suggest that the relation be understood as holding between worlds. The notion of a possible world was simply absent. (Montague mentions Carnap's 1946 interpretation in terms of state descriptions only to reformulate Carnap's account in terms of models.) Montague's 1955 theory is probably best regarded not as an early example of possible worlds semantics as such, but simply as an extension of Tarski's model theory to a language containing modal operators.[16]

11. Meredith and Prior 1956, and Geach 1960

In 1956 Prior took a twelve-month leave of absence from Canterbury University College, New Zealand. Most of the leave was spent in Oxford, where he delivered the John Locke Lectures for that year. Carew Meredith and Prior first met in the summer of 1956, spending time together in Oxford and in Dublin.

Meredith was born into a distinguished Dublin family in 1904. He read mathematics at Trinity College, Cambridge, gaining a Distinction in the Tripos in 1924 (Meredith, D., 1977: 513). Meredith remained in Cambridge, coaching undergraduates, until 1943, when he was appointed lecturer in mathematics at Trinity College, Dublin. He taught there until his retirement in 1964. Meredith had developed an interest in logic as an undergraduate. However, it was not until Łukasiewicz took up residence in Dublin that Meredith was moved to conduct serious research in logic. Meredith attended Łukasiewicz's lectures on mathematical logic at the Royal Irish Academy in 1946 (Łukasiewicz, 1951: 324). Thereafter the two worked extensively together. Łukasiewicz soon adopted Meredith's notation for indicating substitutions. In 1951 there began a stream of publications by Meredith developing aspects of Łukasiewicz's work.

It was probably Łukasiewicz who first introduced Meredith to modal logic. The two worked to extend the use of δ-variables to modal calculi. The axiomatisation given by Łukasiewicz (1953) contained an axiom that Meredith had proposed during their earlier investigation of non-modal propositional logic, $\delta p \rightarrow ((\delta - p) \rightarrow q)$. Substituting '$\Diamond$' for the variable '$\delta$' produces $\Diamond p \rightarrow (\Diamond - p \rightarrow q)$. Perhaps Łukasiewicz did not find this implausible, but Meredith sought improvement, devising a system with δ-variables whose δ-free portion is S5. This work went unpublished at the time and was later incorporated into Meredith and Prior (1965).

Meredith did not publish readily, requiring much encouragement to do so. He was greatly affected by the death of Łukasiewicz in February 1956, saying in a letter to Prior that the event put him 'in a big depression'. (The correspondence between the two had begun in 1952, when at Łukasiewicz's suggestion Meredith wrote to Prior in New Zealand, enclosing a copy of his paper on the extended propositional calculus (Meredith, 1951).) Had it not been for Prior's efforts, it is probable that Meredith would have published nothing more after Łukasiewicz's death. Prior gathered up much of Meredith's unpublished work, often pithy in the extreme, and added his own extensive editorial commentaries. The initial result of this was a manuscript of over 200 pages entitled 'Computations and Speculations'. Two versions of this are among Prior's papers in the Bodleian Library, Oxford. The later version, henceforward 'CS', appears to have been typed in 1961 or 1962. At first Prior planned to publish this work in the form of a book and submitted the MS to Oxford University Press. Thereafter, portions of the MS formed the basis of several joint papers. The rest is unpublished. Following Prior's death in 1969 Meredith published nothing further.

In his papers 'Possible Worlds' and 'Tense Logic and the Continuity of Time' (1962a, b) Prior presented possible worlds semantics for K, M, S4, S4.2, S4.3, B, S5, and other propositional modal systems. Prior gave the impression that the material he set out represented an independent development of possible worlds semantics, due to Meredith and himself (there was no mention of Kripke's paper of 1959, 'A Completeness Theorem in Modal Logic' (Kripke, 1959a).) Investigation of unpublished material by Meredith and Prior has confirmed that this is indeed the case.[17] In 1956 Prior wrote up a brief joint paper entitled 'Interpretations of Different Modal Logics in the "Property Calculus"' (Meredith and Prior, 1956). He circulated it in mimeograph form. This paper contains the essential elements of the possible worlds semantics for propositional modal logic.

Łukasiewicz had drawn Meredith's attention to a paper by Wajsberg – a former colleague of Łukasiewicz's in Poland – which contains an extended calculus of classes equivalent to the modal logic S5 (Wajsberg, 1933). Wajsberg simulated the necessity operator in his calculus by means of an expression $|X|$; this notation, originally introduced by Hilbert and Ackermann, indicates that the predicate X 'applies to all objects' (Hilbert and Ackermann, 1928, Ch. 2; Wajsberg, 1933: 114–115). Prior recorded that Meredith reformulated Wajsberg's calculus to his own satisfaction, calling the result the *property calculus* (CS: 120; Prior, 1967a: 42). (The property calculus was mentioned by Meredith in a note written as early as 1953; see Meredith and Prior, 1965: 102.) Meredith reinterpreted propositions as properties of certain 'objects', a, b, c, etc. He expressed these properties by means of monadic predicates. Meredith wrote '$p(a)$' (or sometimes 'pa') to symbolise the assertion that object a has property p. Truth-functions of propositions became property-forming functions of properties. He treated the modal proposition $\Box p$ as the assertion that p is a property of every 'object'. This assertion, $\forall x p(x)$, and its existential counterpart, were treated as properties possessed either by all objects or by none.

The similarity with Prior's procedure in his Wellington address is marked: Prior treated propositions as properties of times, Meredith as properties of certain objects. Prior read his 'pt' as '*p at t*'; one might suggestively read Meredith's '$p(a)$' as '*p in a*'. Whether Prior had sent Meredith the text of his Wellington address is not known; quite possibly the two had arrived independently at what is essentially one and the same idea.

The joint note of 1956 (which is just one page in length) extended Meredith's property calculus with a binary relation U. Meredith and Prior translated $(\Box p)x$ and $(\Diamond p)x$ by means of expressions of the U-calculus that to the modern eye look very familiar:

$$(\Box p)x = \forall y(\mathrm{U}xy \to py),$$
$$(\Diamond p)x = \exists y(\mathrm{U}xy \mathbin{\&} py).$$

Their extended property calculus consisted of the axioms and rules of ordinary quantification theory supplemented by these definitions, together with certain axioms governing the relation U, and the following clauses:

$$(-p)x = -(px),$$
$$(p \to q)x = (px) \to (qx).$$

It is implicit in the paper that the provable statements of the calculus are those that can be shown to hold of any arbitrarily selected object.

Meredith and Prior listed various axioms for U and they established the following: the K principle $\Box(p \to q) \to (\Box p \to \Box q)$ is provable given the axioms of quantification theory alone; the M principle $\Box p \to p$ is provable given $\forall x\mathrm{U}xx$ (reflexivity); the S4 reduction axiom $\Box p \to \Box\Box p$ is provable given $\forall x\forall y\forall z(\mathrm{U}xy \to (\mathrm{U}yz \to \mathrm{U}xz))$ (transitivity); and $\Diamond\Box p \to \Box p$ (equivalent to the S5 reduction axiom $\Diamond p \to \Box\Diamond p$) is provable given transitivity and $\forall x\forall y(\mathrm{U}xy \to \mathrm{U}yx)$ (symmetry).

The 1956 paper bears the attribution ‘C.A.M., August 1956; recorded and expanded A.N.P.’. It appears that what Prior ‘expanded’ was a note in Meredith’s hand consisting simply of eight lines of symbols. In it Meredith set down the reflexivity and transitivity axioms for ‘U’, the above clauses for negation and implication, and the above definition of ‘$\Box$’. He listed the theses $\Box(p \to q) \to (\Box p \to \Box q)$, $\Box p \to p$ and $\Box p \to \Box\Box p$, and he marked $\Diamond p \to \Box\Diamond p$ with ‘$\dashv$’. This symbol, reversing Frege’s assertion sign, was taken over from Ivo Thomas by Meredith and Łukasiewicz to represent the rejection of the formula that follows it (CS: 94).

The Meredith–Prior paper is purely formal and no philosophical explanation is offered, either of the U-relation or of the nature of the objects to which the variables of the calculus refer. Thus the question arises whether Meredith and Prior themselves regarded the variables of their calculus as referring to possible worlds. It is certainly the case that Prior was thinking this way by the time he wrote his paper ‘Possible Worlds’ (1962a) and the section entitled ‘The Logic of World-Accessibility’ of his paper (1962b). He said:

> Suppose we have the usual variables p, q, r, etc., for statements, and a, b, c, etc., for names of ‘worlds’ or total states of affairs. Let us then write pa for ‘It is the case in world a that p’. (1962b: 140)

But that was later, and in any case Prior was by that time familiar with Kripke’s paper of 1959. (Prior had refereed the paper for the *Journal of*

Symbolic Logic.) Were Meredith and Prior perhaps the discoverers, like Wajsberg, merely of an interesting formal analogy between certain modal calculi and certain theories in extensional logic? As Kripke has remarked, if Meredith and Prior had possessed the possible worlds interpretation of their formalism in 1956, Prior could very easily have included in their note a brief comment to that effect (such as the remark quoted above), but he did not do so. Moreover, there is a sentence in Prior's book *Past, Present and Future* that can be interpreted as saying that it was Geach who several years later first suggested to Prior that the letters a, b, c, etc., of the calculus might be taken to name possible worlds.[18] Kripke observes on the basis of this evidence that it seems fair to conclude that the possible worlds interpretation was unknown to Meredith and Prior in 1956.[19]

Further evidence has now come to light which appears to settle the matter conclusively. Meredith and Prior did possess the possible worlds interpretation of their calculus in 1956. Indeed, Meredith – strongly influenced by the *Tractatus* – was expounding a sophisticated theory of possible worlds at that time. There survives a letter to Prior dated 10 October 1956 in which Meredith uses the term 'possible world'. In a passage concerning the use of matrices to characterise some of his modal systems, Meredith remarked that 'one other possible world' is required in order to refute a certain formula. Commenting on this in CS, Prior explained that the

> 2^m values of the matrices ... are interpretable as distributions of the ordinary two truth-values in m 'possible worlds'. (CS: 119)

Meredith would write 111 . . . 11, 111 . . . 10, . . . , 000 . . . 00, for the 2^m values 'true in all m worlds', 'true in all but the mth', . . ., 'true in no world' (CS: 119). The sequences that contain '1' in one and only one place represent propositions that Meredith also referred to as 'worlds' (quipping that each world is 'next to impossibility'). The idea is that a sequence in which '1' occurs at only the ith place represents a proposition p_i equivalent to the conjunction of all propositions true in the ith possible world. Prior termed these propositions 'world-propositions' (Meredith and Prior, 1965: 102). A world-proposition is true at one and only one world; and if any world-proposition is conjoined with some proposition not implied by it, the result is an impossible proposition. In terms of ontology, Meredith was a reductionist about worlds: he identified the ith world with the world-proposition p_i. Prior recorded that Meredith preferred 'to explain the notion of a "possible world" in terms of particular things being possible, rather than vice versa' (Prior, 1967b: 11). Elsewhere Prior recalled:

> I remember ... C. A. Meredith remarking in 1956 that he thought the only genuine individuals were 'worlds', i.e. propositions expressing total world-states, as in the opening of Wittgenstein's *Tractatus* ('The world is everything that is the case'). (Prior, 1968: 141)

On Meredith's account, a possible world is any proposition sufficiently comprehensive as to either imply or be inconsistent with any given proposition, and yet not so comprehensive as to imply every proposition (Prior, 1967b: 11). Prior observed that Meredith placed the insight

> that the actual world is the most that one can *truly* say and any world is *a* maximum that one can say without contradiction

beside the Wittgensteinian insight that the impossible is the most that one can say ('a contradiction fills the whole of logical space', *Tractatus* 4.463) (CS: 120).

In a note written in 1953 (which forms part of Meredith and Prior, 1965) Meredith singled out one world-proposition and added it to the language of his modal calculus in the form of a propositional constant n. In a letter written in 1956 he called n 'the world' (Meredith to Prior, 10 October). n is:

> 'the world' in the sense of Proposition 1 of Wittgenstein's *Tractatus*, i.e. 'everything that is the case'. One can think of this as a conjunction of all truths, or perhaps of all 'atomic' truths. (CS: 117)

The following axioms concerning n were added to the modal calculus (letter from Meredith to Prior of 10 October 1956, CS: 117–118, Meredith and Prior, 1965: 103). (1) n is itself an axiom: the world is the case. (2) $\Box n \rightarrow p$. This axiom says that n is contingent (for there are contingent truths, and any conjunction with contingent components is itself contingent). One way of expressing n's contingency is to say that anything whatsoever is implied by the statement that n is necessary. (3) $p \rightarrow \Box(n \rightarrow p)$. n entails each true statement; or, more picturesquely, the world is everything that is the case.

Of particular interest is Meredith's treatment of his constant 'the world' in the property calculus. (This treatment dates from at least as early as 1953; see Meredith and Prior, 1965: 99, 103.) The proposition 'n' is represented in the calculus by the property of being identical with a selected object, designated 'a' (CS: 120, Meredith and Prior, 1965: 103). Formally:

$$nx \quad = \quad x = a.$$

That is to say, n is a property of x just in case x is the actual world. Meredith took formulae that express properties of a, as well as formulae that express a property of every object, to be theorems of the calculus. Prior comments:

> This is analogous to the use of matrices in which the value n, or 'true in n only', is designated as well as the value 'true in all worlds'. (CS: 121)

Meredith's introduction of the constant 'the world' into his modal calculus was motivated by a concern of Łukasiewicz's. It is a peculiarity of many modal systems that their asserted formulae all remain theorems if 'Necessarily p' and 'Possibly p' are equated with the simple 'p'. For example, the principles $\Box p \rightarrow p$ and $p \rightarrow \Diamond p$ both collapse under this interpretation to the law of identity $p \rightarrow p$. It may be suggested that such a calculus is nothing more than ordinary propositional logic in disguise. Łukasiewicz therefore desired a modal calculus that does not coincide with ordinary propositional logic under this identification. To that end he included, besides assertions, 'rejections' among the axioms of his calculus, employing Thomas's reversed assertion sign to indicate rejection. For example, he took the rejections $\dashv \Diamond p \rightarrow p$ and $\dashv p \rightarrow \Box p$ as axioms. Prior argues that this is not much help: the identification of $\Box p$ and $\Diamond p$ with p then leaves what is merely an inconsistent propositional calculus, in which the law of identity is both asserted and rejected (CS: 116). Meredith took a different tack:

his principal device for removing any suggestion of triviality from his modal logic is the introduction of a contingently true propositional constant ... 'the world'. (CS: 116–117)

The resulting modal calculus has the property that not all its theorems remain provable if $\Box p$ and $\Diamond p$ are identified with p. An example is the axiom $\Box n \rightarrow p$. The result of replacing $\Box n$ by n, $n \rightarrow p$, which says that the world is inconsistent, is not provable.

Meredith had no interpretation to offer of his U-relation. Nor did Prior. In 1960 Geach coincidentally wrote to Prior providing just such an interpretation. Geach himself knew nothing of the property calculus.[20] (Nor was he, at that time, familiar with Kripke (1959a, b).) He drew his inspiration from science fiction. The letter read:

Here's a thing that might amuse you, since you combine an interest in modal logic and in SF. I am sure you are very familiar with the SF stories in which there are parallel worlds that you can reach by machine. It occurs to me that the systems between S4 and S5 (inclusive) can all be characterised in terms of different suppositions as to the possibilities of world-jumping.
L = 'in the world we are in, and in every world we can jump to from it, it is the case that ... '.
M = 'in the world we are in, or in some world we can jump to from it, it is the case that ... '.
Then the S4-reduction just means that my being able to jump to a world that either is the world W, or is a world I can jump from to W, is to count as being able to jump to W. The S5-reduction means that I can jump from any world to any world. Other hypotheses as to my jumping abilities, e.g. that I can jump from W′ to W if I can jump from W to W′, would give intermediate logics. I think this model may be useful. (Geach to Prior, 15 April I960)[21]

Geach used the term 'Trans World Airlines' for this voyaging between worlds. He remarks that he at no time took this formalism seriously, attaching no ontological significance to the idea of parallel worlds. These were a useful imaginative device, nothing more.[22]

Prior seized on Geach's idea and took to describing U as a relation of *accessibility* between worlds. He wrote:

> Let us ... interpret (Lp)a, 'Necessarily-p in world a', as short for ... 'p is true in a and in all worlds accessible from it'. (1962b: 140; see also 1962a: 36–7)

Lemmon, in a draft of material intended for his and Dana Scott's projected book 'Intensional Logic', mistakenly credited Geach with the idea that the binary relation 'may be intuitively thought of as a relation between possible worlds'. In a letter to Scott, written after Lemmon's death in 1966, Prior states: 'What Geach contributed was not the interpretation of [U] as a relation between worlds ... but the interpretation of [U] as a relation of accessibility'. (Both quotations are from Segerberg, 1977: 25.)

Prior's paper 'Possible Worlds' and the section entitled 'The Logic of World-Accessibility' of his (1962b) are a glorious amalgam of ideas due to Meredith, Geach, and himself. These papers are neglected classics of modal logic.

12. Smiley, 1955–1957

In his doctorial thesis (Cambridge, 1955), Smiley pursued an approach similar to McKinsey's 'syntactical' account of possibility, but independently of McKinsey's work.[23] (Smiley did not learn of McKinsey (1945) until 1958 or 1959.) In Smiley's translational semantics, the truth-conditions of $\Diamond A$ are those of a disjunction of A with a variable number of translations, i.e. formulae of the same form as A having different atomic components.[24] Smiley proved the completeness of the system M relative to this semantics.

Smiley subsequently extended the semantics to S4 and S5 by imposing further conditions on the admissible translations: closure for S4, and closure plus the existence of an inverse for S5 (these conditions Smiley discovered independently). In a typewritten lecture handout (Smiley, 1957), he stated that the method used in his doctoral thesis to prove the completeness of M (namely, reduction to normal form and induction on the modal degree of formulae) can be extended to prove completeness for S4 and S5.[25] In the same handout, Smiley suggested that the translational semantics be understood in terms of truth in possible worlds. He wrote:

The operations $(\ )_{\underline{i}}$ are interpreted as translations of the descriptive or non-logical terms involved in the proposition, including an identity transformation $\underline{A}_0 = \underline{A}$. Then in all of S, S4, S5, $1\underline{A}$ may be read as $(\underline{i})A_{\underline{i}}$ and $-1-\underline{A}$ as $(\underline{Ei})\underline{A}_{\underline{i}}$: that is, necessary truths are true in all ‘possible worlds’, and a proposition is possible if it is true in some ‘possible world’. Choice between the three systems, or others, would be made on acceptance of the stipulations governing the translations $(\ )_{\underline{i}}$ – e.g. whether ‘possible worlds’ are to be arranged in a hierarchy of degrees of remoteness from the actual world or not. (Smiley, 1957)

Smiley seems to have been the first to announce completeness proofs for propositional M, S4 and S5 relative to a semantics explicitly interpreted in terms of the notion of a possible world. Smiley remarks:

It is a pity that [translational semantics] turns out to be a wrong turning outside the realm of propositional logic. ... Translational semantics is incapable of dealing with variations in the size of the domain.[26]

13. Kanger, 1957

Kanger’s ‘general theory of modalities’ was published in 1957 by the University of Stockholm under the title ‘Provability in Logic’. In the preface, Kanger notes that ‘[m]ost of the investigations contained in this essay were made for a course in logic, which I gave during the spring term of 1955 at the University of Stockholm’. His booklet – really little more than a lecture guide – is difficult to read, and at the time his work was by and large overlooked.

The nature of the important contribution made to modal semantics by Kanger in this booklet has been distorted. Kaplan’s statement that ‘Kanger ... introduced a relation between worlds and explicitly stated completeness theorems for M, S4 and S5’ goes well beyond anything to be found in Kanger’s own writings (Kaplan, 1966: 122). Føllesdal has written, apparently without knowledge of Montague’s lecture at UCLA in May 1955:

Kanger [1957a] proposed the first fully model theoretic interpretation of modal logic. (Føllesdal, 1994: 886)

Føllesdal continues:

Moreover, he [Kanger] introduced a fundamental new idea, which ... at that time was an innovation: ... Kanger regarded the notion of a possible world as a *relative* notion. One world may be possible relative to some other worlds, and not possible relative to further worlds. (ibid.: 886)[27]

In fact, the notion of a possible world is nowhere to be found in Kanger’s booklet. While Kanger did state a soundness theorem for the systems M, S4 and S5 (1957a: 40), he stated no modal completeness theorems and his work left questions of completeness open, even in the case of propositional modal systems.

Kanger pursued a model theoretic approach. For each of his one-place modal operators, M_i, Kanger introduced a quaternary relation R_i. As with Montague's contemporaneous work, Kanger's relation holds between models rather than points or indices.[28] Kanger offered no philosophical interpretation of the relation R_i nor of his models. He gave conditions for the truth in a structure of formulae of the form $M_i A$, in terms of A's being true in all structures related to that structure by R_i. Kanger listed various possible conditions that R_i might satisfy, equivalent to the binary relation being reflexive, transitive, or symmetrical. He also gave a condition validating the Barcan formula and its converse. Kanger referred in a footnote (1957a: 39) to Jónsson and Tarski (1951), from whom his use of the relational apparatus presumably derived.

The notion of a possible world is not to be found in Kanger's 1957 booklet, but is there evidence elsewhere in Kanger's writings from this period to indicate that he may have been considering a possible worlds analysis of the modalities? Lindström suggests not:

> Nor is there in his [Kanger's] early works from 1957 any mention or discussion of possibilia (possible worlds, counterfactual states of affairs, possible individuals). I do not think this is an accident. . . . In (1957b), he explicitly mentions it as an advantage of his approach that it does not presuppose any 'intensional' entities like Fregean senses, meanings or intensions. Although he does not discuss the matter, one gets the decided impression that Kanger's ontology is no more hospitable toward possibilia than it is toward intensional entities. (1998: 204)

However, in correspondence Kanger stated:

> As far as the term 'possible world' is concerned, I don't know who used it first. However, in . . . 'New Foundations for Ethical Theory', Stockholm, December 1957 (reprinted in Hilpinen . . . 1971 [sic]) . . . an alternative universe in deontic logic is occasionally referred to as 'a moral standard for our universe' (cf. Hilpinen, p. 57).[29] (Kanger to Routley, 11 September 1981)

The page to which Kanger refers contains this statement:

> assumption I of the preceding section . . . implies, roughly speaking, that there is a universe r' which is a moral standard for our universe r^*. (Hilpinen, 1970: 57)

Earlier in 'New Foundations for Ethical Theory', having introduced the term 'range' for a non-empty class of individuals (ibid.: 44), Kanger said:

> A judgement . . . is *analytic* if [it] holds in every range. (ibid.: 48)

He explained:

Our definition of analyticity may be regarded as an explication (and an extension to imperatives) of the idea that an analytic proposition is a proposition that is true in every possible universe. (ibid.: 49)

It seems, therefore, far from true that in 1957 Kanger was inhospitable toward the introduction of possible worlds. Nevertheless, Kanger appears to have made no systematic attempt to connect the formal model theory set out in his 1957 booklet with his notion of a possible universe.[30]

14. Hintikka, 1957–1961

In work on deontic logic in 1957, Hintikka independently introduced the concept of a binary relation holding between states-of-affairs, both possible and actual. In 'Quantifiers in Deontic Logic' he wrote:

What do we mean by saying that f is permitted? ... When speaking of permissions, we are not really speaking of the actual state of affairs at all. ... [W]e are saying that a state of affairs different from the actual one is consistently thinkable, viz. a state of affairs in which f is done but in which all the obligations are nevertheless fulfilled. ... [The set of formulae] μ was thought of as being concerned with the actual state of affairs. ... [W]e must consider, in addition to μ, another set [of formulae] μ^* related to μ in a certain way. This relation will be expressed by saying that μ^* is *copermissible with* μ. We may think of μ^* as being concerned with the (imagined) state of affairs in which f was supposed to take place. (1957a: 11)

Hintikka's semantical treatment of the permissibility operator P was:

If $Pf \; \varepsilon \; \mu$ then there is a set μ^* copermissible with μ such that $f \; \varepsilon \; \mu^*$. (ibid.: 11)

He developed an account of satisfiability whereby a set of formulae is satisfiable if and only if it can be imbedded in a model set, i.e. a set satisfying certain semantical conditions, including the one just stated (ibid.: 10). A formula f is valid if and only if $\{-f\}$ is not satisfiable (ibid.: 13). Hintikka exhibited a method of establishing that sets of formulae are not satisfiable, and hence of showing that individual formulae are valid (ibid.: 13–15). He described the copermissibilty semantics as deriving 'from an earlier treatment of quantification theory [Hintikka, 1955a] along the same lines as well as from a similar (unpublished) theory of modal logic', and he remarked that most of the formal considerations in 'Quantifiers in Deontic Logic' were 'special cases of this new general theory of modal logic I have developed' (ibid.: 10).

'Quantifiers in Deontic Logic' contained no conditions on the copermissibility relation. However, Hintikka recalls that he came to the idea of achieving different modellings by imposing different conditions on the binary relation 'at around the same time' that he introduced the copermissibility concept, and that in the winter or spring of 1957 he was 'aware of the possibility of, and had a vague idea of, a completeness proof' (although it was to be some time before he wrote anything down).[31] In the alethic case, he thought of the binary relation as being a relation of relative possibility holding between possible worlds. His thinking on modal logic was guided 'from the beginning' by the idea of possible worlds: his model sets formed 'a syntax for possible worlds'.

Hintikka gave the first public presentation of his possible worlds semantics for modal logic in a short series of informal talks in the Boston area (the seminars were organised by Hartley Rogers).[32] Hintikka held a Junior Fellowship at Harvard from 1956–1959. He remembers that the seminars took place either during the first half of 1958 or during the academic year 1958–59. In these seminars Hintikka gave completeness proofs for versions of M, S4 and S5 with quantifiers, invoking the now standard conditions on what he termed an *alternativeness* relation between possible worlds. (Noticing a minor technical flaw in his method for proving completeness, Hintikka was able to correct the error in the interval between two of the seminars.) This work was a continuation of his work on the semantics of the first-order predicate calculus (Hintikka, 1955a, 1955b), and his modal completeness proofs were variants of his 1955 completeness proof for predicate calculus. Hintikka observes that completeness in the modal cases was 'obvious' given his earlier treatment of the predicate calculus in terms of model sets. Unfortunately, Hintikka's extensive notes of the material presented in the seminars are now lost, and in their absence it is difficult to determine the exact form that his completeness proofs took. The rules for model set construction correspond to proof rules in the style of Beth tableaux, or can be inverted to produce something like Gentzen-style sequents. Hintikka believes that the proofs he gave in the seminars probably proceeded in terms of something like the tableau method. He is certain that variable domains formed part of the treatment, together with the idea that the domain of a quantifier might consist of actual individuals, or of individuals from alternative worlds, depending on the location of the quantifier in the formula. The corresponding proof-rules for the quantifiers contained existential presuppositions. Hintikka recalls that systems with and without the Barcan formula were considered in the seminars, and that he showed the Barcan formula to be valid only in the presence of a principle laying down the transfer of existence assumptions from alternatives to the worlds to which they are alternative.

Hintikka's 1957 paper 'Modality as Referential Multiplicity' gives something of the flavour of his 'unpublished essay on the foundations of modal logic, some features of which occur in my paper on "Quantifiers in deontic logic"' (1957b: 53):

> The standard way of treating the logic of quantification in the spirit of the theory of reference is by means of the notion of a model. I have discussed this notion elsewhere [Hintikka, 1955a] and shewn that it can be replaced by the slightly more flexible notion of a model set of logical formulae. It turns out that an intuitive and powerful theory of modal logic can be based on these notions. The main novelty is that we have to consider several interrelated models (or model sets). They correspond to the different situations we want to consider in modal logic, and they are interconnected, in the first place, by a rule saying (roughly) that whatever is necessarily true in the actual state of affairs must be (simply) true in all the alternative states of affairs. (1957b: 61–62)

In the same paper (1957b: 53–54) Hintikka stated that the unpublished essay contained a treatment of identity in modal logic in which the necessity of identity was rejected, saying:

> Except in a completely deterministic universe, the fact that something *is* does not in general imply that it *could not have been* otherwise; and I cannot see any reason for making an exception in the case of identities.

There he also argued that terms are to be treated referentially when 'modal operators mix with quantifiers', invoking multiple reference-relations and an epistemic treatment in order to deal with the difficulties of opacity (1957b: 54–61).

Hintikka spoke on the semantics of quantified modal logic at a conference at Stanford in 1960, with an abstract appearing in the mimeographed proceedings. A fuller treatment appeared in 1961 under the title 'Modality and Quantification' (1961b). There Hintikka explained his notion of a model set in terms of Carnap's state-descriptions:

> A set of formulae λ is satisfiable if and only if there is a state-description in which all the members of λ hold. . . . [A] set of formulae μ is *the* set of *all* formulae which hold in some particular state-description if and only if it satisfies the following conditions [concerning negation, identity, conjunction, disjunction, and the two quantifiers]. I shall call a set of formulae which satisfies [these conditions] a *model set* . . . [A] model set is the formal counterpart to a partial description of a possible state of affairs (of a 'possible world'). . . . In discussing notions like possibility and necessity, we have to consider what happens in states of affairs different from the actual one. In our definition of satisfiability, we therefore have to consider *sets* of model sets. Such sets of sets we shall call *model systems*. (1961: 119–122)

Hintikka gave the following clause for the modal operator $\Box$, where Ω is a model system:

> If $\Box A \;\varepsilon\; \mu \;\varepsilon\; \Omega$ and if $v \;\varepsilon\; \Omega$ is an alternative to μ, then $f \;\varepsilon\; v$. (ibid.: 123)

Here Hintikka stated (but did not prove) soundness and completeness results for propositional M, S4, B and S5, and gave an extensive discussion of 'the problems which arise when modality is combined with quantification (and/or identity)' (ibid.: 124). He presented a version of the semantics in which substitutivity of identicals and necessity of identity hold and another in which these fail (ibid.: 127–8).

15. GUILLAUME, 1958

In two notes presented to the Académie des Sciences in February 1958, Guillaume generalised Beth's method of semantic tableaux to yield tree formulations of M and S4. Guillaume showed that a formula has a closed S4-tree (M-tree) if and only if the formula is provable in S4 (M). Guillaume's approach was topological. He referred to the work of McKinsey and Tarski, and of Kanger.

16. BINKLEY, 1958

At Duke University in 1958 (or possibly late 1957) Binkley developed an approach that he called 'world theory', which reduced modal logic to quantification over possible worlds. Binkley says:

> Using the equation 'Possibly *p*' = 'There is a world in which *p* is true', I would simply translate modal statements into statements of predicate logic, and then employ the techniques of that logic . . . One way to do this was to add another term to all predicates, so that '*x* is red' becomes '*x* is red in *w*', name the actual world '*a*', help oneself to a domain of possible worlds, and lay down various axioms about them.[33]

Binkley describes his influences as having been 'von Wright for the quantification, Leibniz for the possible worlds'.[34] Some years later, under the influence of Hintikka, he added a relation of alternativeness between worlds. Binkley's main interest lay in decision methods for modal logic; world theory opened the possibility of recruiting decision methods from predicate logic, such as resolution, to this end (Binkley and Clark, 1967, 1968; Snyder, 1971).

17. BAYART, 1958–1959

In 1958, Bayart published his paper 'Soundness of First and Second Order S5 Modal Logic', followed, in 1959, by 'Quasi-Completeness of Second-Order S5 Modal Logic and Completeness of First-Order S5 Modal Logic'.

These papers represent an independent formulation of possible worlds semantics which shares features with Kripke (1959a). Bayart published in French and his work is little-known.

Bayart (1958) begins:

In order to formulate a semantic theory of modal logic it is not enough to define, for example, the necessary as that which is true in all models and the possible as that which is true in one model. These definitions only serve to introduce the notions 'necessary' and 'possible' into the metalanguage. A semantics of modal logic requires that we give an object-language containing symbols for modalities and that we define in which conditions we will attribute the values 'true' or 'false' to the formulae of this object-language. ... It is a theory of this type that we propose to develop in the present article, in which we have been inspired by the Leibnizian definition of the necessary as being that which is true in all possible worlds. It is not, in our understanding, the task of the logician to examine the value of this Leibnizian metaphysic. We can limit ourselves to noting that in being inspired by this metaphysic one can formulate for modal logic S5 a semantic theory analogous to the semantic theory traditionally formulated for non-modal logic.[35]

What Bayart (1958) called a 'universe' consists of two non-empty sets having no common elements, a set of individuals and a 'set of worlds'. A 'system of values relative to a universe U' assigns an individual to each individual variable and an 'n-place intensional predicate defined from universe U' to each n-place predicate variable. An n-place intensional predicate is a function that at each world in U assigns either *True* or *False* to each n-tuple of individuals from U. Where U is any universe, M any world of U, and S any system of values relative to U, Bayart defined 'true for *UMS*' and 'false for *UMS*' by a recursion of the now familiar sort. The clauses for $\Box$ and $\Diamond$ are:

$\Box p$ is true for *UMS* if for every world M' of U, p is true for $UM'S$; otherwise $\Box p$ is false for *UMS*.
$\Diamond p$ is true for *UMS* if there is a world M' of U such that p is true for $UM'S$; otherwise $\Diamond p$ is false for *UMS*.

A proposition is valid in *UM* if and only if, for every system of values S relative to U, p is true for *UMS*; p is valid in U if and only if, for every world M of U, p is valid in *UM*; and p is valid if and only if p is valid in every universe.

Bayart (1958) gave Gentzen-style sequent-calculus formulations of first-order and second-order quantified S5. Extending the definitions of 'valid in *UM*', etc., to apply also to sequents, he proved that both the first-order and the second-order system is sound, in the sense that only valid sequents are deducible.

Bayart ended this paper by considering what happens if the index-sets of possible worlds are omitted from the semantics. He wrote:

One can be tempted to mix up the notions of necessity and validity. One would then be led to formulate the following semantic theory: Instead of giving a universe composed of a set of individuals and a set of worlds, one would limit oneself to giving a domain *D*, that is, a set of individuals.

Bayart defined, in the obvious way, the notions of a system of values *S*, true for *DS*, valid for *D* (true for *DS* for every system of values *S*), and valid (valid for *D* for every domain *D*). In the recursive definition of 'true for *DS*', the clause for $\Box$ is:

$\Box p$ is true for *DS* if for every system of values S', p is true for DS'; otherwise $\Box p$ is false for *DS*.

Bayart showed that:

[t]he semantic rules that we have just formulated render S5 unsound.

He gave an example of a formula that is deducible in the first-order system but which is not valid according to the modified semantics:

$$\exists y \Box (Fx \vee -Fy).$$

He concluded:

modal logic does not identify the notions of necessity and validity.

Bayart (1959) proved that first-order quantified S5 is complete relative to his possible worlds semantics. He employed a Henkin-style proof (modestly remarking that his 'exposition is only an adaptation of Henkin's theorem to S5 modal logic' (1959: 100)). There is no corresponding completeness result to be had in the second-order case, since (as Bayart noted) the set of valid formulae of second-order non-modal logic is not recursively enumerable. Bayart showed that second-order quantified S5 is *quasi-complete* in Henkin's sense.

Bayart did not discuss modal systems weaker than S5 and the binary relation did not appear in his semantics.

18. Drake, 1959–1961

In 1959 Drake began his doctoral studies with Smiley at Cambridge.[36] Smiley drew Drake's attention to McKinsey (1945). Drake refined McKinsey's translational semantics and was able to prove completeness theorems for propositional M, S4 and S5, using semantic tableaux (Drake, 1962).[37] (McKinsey had established only soundness results in his (1945).) Working firmly in the algebraic tradition, Drake employed the functional conditions of identity, closure, and inverse. He made no connection between his translational semantics and possible worlds (although he was aware of Kripke (1959a)). Soon, however, the algebraic tradition was eclipsed by the model-theoretic approach. Drake says:

> I certainly regarded Kripke's later work as superseding mine, and even more so the work of Dana Scott which I learned of in about 1965. I can remember being glad to have my PhD in my pocket when I saw that work.

19. Kripke, 1958–1965

Kripke first became interested in modal logic in 1956, as a result of reading Prior's paper 'Modality and Quantification in S5'.[38] Kripke was at this time still at high school, working on logic in almost complete isolation in Omaha, Nebraska. His first paper on possible worlds semantics, 'A Completeness Theorem in Modal Logic', was submitted in 1958 and published in 1959 (Kripke, 1959a). (The date shown in the journal for submission, 25 August 1958, was in fact the date of submission of the final MS incorporating the revisions requested by the referee (Prior). Kripke believes that the original submission was probably in March 1958 and at any rate some time in the spring of that year.) Here Kripke stated and proved a completeness theorem for an extension of S5 with quantifiers and identity; proof was by means of an adaptation to modal logic of Beth's semantic tableaux. The paper did not discuss systems weaker than S5 (save for a brief mention in the closing sentences), nor variable domains, and the binary relation made no appearance.

Kripke's first publication to mention the binary relation – which he interprets as a relation of relative possibility between worlds – was written in 1962 and appeared in 1963 (Kripke, 1963a). Kripke proved completeness for propositional M, S4, B, and S5. In another paper published in 1963, Kripke gave axiomatisations of quantified extensions of M, S4, B, and S5 in which (unlike his 1959 axiomatisation of quantified S5) only *closed* formulae are asserted (1963b: 69). He discussed variable domains and gave countermodels in quantified S5 to both the Barcan formula and its converse (ibid.: 67–68). In 1965 Kripke published completeness results for the systems $S2^n$, S3, S6, S7, S8, E2, E3, D2 and D3.

Kripke reports that the idea of the binary relation occurred to him much earlier than 1962, in fact shortly after his paper on S5 was first submitted in the spring of 1958. Certainly by the late summer of that year, Kripke had a completeness result for S4. On 3 September 1958 he wrote to Prior, mentioning his work on semantical completeness theorems for quantified extensions of S4 (with and without the Barcan formula). In the letter, Kripke gives a branching-time matrix, characteristic for S4. This is essentially a tense-logical interpretation of the reflexivity + transitivity semantics for S4.[39]

In the autumn of 1958, Kripke became an undergraduate at Harvard. Some of his teachers there advised him to give up his work on modal logic. This discouragement accounted in part for the long delay between the publication of his completeness theorem for S5 and his results for other systems. He announced his additional results in an abstract published in December 1959 (Kripke, 1959b) and by means of a brief addendum to his first paper. The abstract stated completeness results for a wide range of propositional modal and deontic systems, saying that '[q]uantifiers can be added, with completeness theorems preserved' and that the Barcan formula 'turns out to hold when there are no "possible existents" beyond the individuals of the real world' (ibid.: 324). In the abstract, Kripke noted: 'For systems based on S4, S5, and M, similar work has been done independently and at an earlier date by K. J. J. Hintikka' (ibid: 324). However, it is not clear which of the two was in fact the first to produce a fully worked out completeness proof (it must have been a matter of a few months at most). Unless the exact date of Hintikka's seminars in Boston (see Section 14) can be discovered, this question may never be settled.

20. The Kripke–Kanger Correspondence, 1958

On 24 January 1958, Kripke wrote to Kanger at the University of Uppsala. Kripke said in the letter that he was writing at the suggestion of H. B. Curry, in order to request reprints of Kanger's 'Provability in Logic' and 'A Note on Partial Postulate Sets for Propositional Logic'.[40] Kripke's request was in connection with work of his relating to formal deducibility and Gentzen-like systems. In a letter written on 11 September 1981, Kanger stated that he sent Kripke 'Provability in Logic' in February 1958.[41] He stated also that he included his papers 'The Morning Star Paradox' and 'A Note on Quantification and Modalities'.[42]

Kanger's 'Provability in Logic' arrived in Omaha during the critical months in which the binary relation first appeared in Kripke's work.[43] 'Provability in Logic' contained relational modellings of the systems M, S4 and S5. Kanger also drew attention to the work of Jónsson and Tarski, saying in a footnote: 'Similar results in the field of Boolean algebras with operators may be found in Jónsson and Tarski 1951' (1957: 39). Kripke's knowledge of 'Provability in Logic' raises the question whether his work was influenced by that of Kanger. There is also the question whether Kripke's work was influenced by that of Jónsson and Tarski, either directly, or indirectly via the influence of Jónsson and Tarski (1951) on Kanger. Concerning these issues, Kripke reports:

As to the ... question of influence, I am rather confident that there was none, or virtually none. As everyone knows, 'Provability in Logic' is written in a complicated way that makes it hard to understand. Although my reason for asking for it was for the other parts, not having to do with modal logic, presumably if I gave this monograph a minimal look from beginning to end I noticed that it had something to do with modal logic. But if so, I got little out of it. I may well have invented my own version of the relational semantics before it arrived. Obviously I later realised that his [Kanger's] work had some relation to mine. I believe I acknowledge it in my papers in 1963 [1963a, note 2; 1963b, note 1].

I definitely did not get my information on the work of Jónsson and Tarski from Kanger's monograph. Why I didn't notice the reference, or if I did notice it, didn't take it up, is obscure to me at this time. However, in 'Semantical Analysis of Modal Logic I', published in 1963 and written in 1962, I refer to Jónsson and Tarski in a footnote [note 2] (actually hastily written since I hadn't read their paper thoroughly when I had to complete my own paper). I call it the most striking anticipation of my work and say that I noticed it only recently.[44]

21. Terminus Initii

1958 and 1959 were marvelous years for possible worlds semantics, with Bayart, Hintikka and Kripke obtaining completeness results for various formulations of quantified modal logic, and Kripke proving the completeness of a large assortment of propositional systems. In the wake of this pioneering work modal logic enjoyed a boom in popularity. The 1960s saw a burgeoning of interest in not only alethic modal logic and philosophical applications of possible worlds semantics, but also tense logic, deontic logic, epistemic and doxastic logic, the logic of action, erotetic logic, relevance and relevant logic, intuitionistic model theory, and (a little later) dynamic logic.[45] Modal logic had come of age.

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of his Fiftieth Birthday' (University of Uppsala Philosophy Department, 1999).

NOTES

[1] A matrix is a finite table of truth-values.

[2] Translation by Fraser White.

[3] In a footnote (1963: 69) Kripke reported having heard that McKinsey, who died in 1953, had left an unpublished model-theoretic account of certain Lewis systems. Smiley recalls Prior, Lemmon and Meredith saying that they had tried to locate this material, but without success (personal communication, 2000).

[4] Smiley in personal communication with the author (2000).

[5] Cocchiarella in personal communication with the author (1993, 1996). Cocchiarella was a student of Montague's at UCLA from about 1958.

[6] The paper built on McKinsey and Tarski (1944) and (1946).

[7] McKinsey's and Tarski's result for S5 was later improved by McKinsey's student Scroggs (while at the Oklahoma Agricultural and Mechanical College), who proved that any sentence not a theorem of S5 is false in some finite normal matrix for the system (Scroggs, 1951); this result is nowadays expressed by saying that S5 has the finite model property. McKinsey had shown that S2 and S4 have this property (McKinsey, 1941).

[8] Kripke in personal communication with the author (1993). (Quoted in Copeland, 1996: 13.)

[9] Translation by Elisabeth Norcliffe.

[10] The manuscript is deposited in the Bodleian Library, Oxford. Oxford University Press had accepted the manuscript for publication on the condition that Prior modify it in various ways, but in the course of complying Prior ended up writing a completely different book, his famous volume *Formal Logic* (Prior, 1955).

[11] The manuscript is among Montague's papers at UCLA. Extracts from the manuscript are quoted by permission of Department of Special Collections, Charles E. Young Research Library, UCLA.

[12] Silver in personal communication with the author (2001). I am grateful to Graham Solomon for putting me in touch with Silver.

[13] This paragraph is based on conversations and correspondence with Kalish (1998). Quotation marks indicate Kalish's words.

[14] Kalish believes that the paper may still exist, 'buried in 40 years of accumulated material'. It was in part the appearance of Kripke's work in 1959 which stirred Montague to publish his 1955 talk. He says in a footnote to the published version that his paper

> contains no results of any great technical interest; I therefore did not initially plan to publish it. But some closely analogous, though not identical, ideas have recently been announced by Stig Kanger (in 'The Morning Star Paradox' [Kanger, 1957b] and 'A Note on Quantification and Modalities' [Kanger, 1957c] and by Saul Kripke (in 'A Completeness Theorem in Modal Logic' [Kripke, 1959a]). In view of this fact, together with the possibility of stimulating further research, it now seems not wholly inappropriate to publish my early contribution. (1960: 269)

[15] Kaplan in personal communication with the author (1998).

[16] David Lewis has said (in personal communication with the author, 1996):

> I don't know about 1955–60, but in later years I think Montague would have opposed taking models as worlds for a familiar technical reason. Two worlds might be just alike in their domains and in the extensions they assigned to predicates – alike *qua* models – yet not alike in their accessibility relations. Likewise, still more obviously, in the case of times; and it tended to be thought that worlds and times would be treated alike. So identifying worlds (or times) would have amounted to imposing a troublesome and unmotivated constraint on model structures.

[17] This material is held in the Bodleian Library, Oxford, and at the University of Canterbury, New Zealand (where Prior was Professor of Philosophy until 1958).

[18] 'It was later suggested by Geach that we might take a, b, c, etc., to name worlds, and Uab to mean that world b is "accessible" from world a' (1967a: 42). As will become clear, Prior did *not* intend this remark to be taken to imply that Geach was the *first* to suggest that a, b, c, etc., name worlds.

[19] Kripke in personal communication with the author (1993).

[20] Geach in personal communication with the author (1996).

[21] In another letter to Prior, dated 3 April 1964, Geach anticipated the counterpart relation. (I am grateful to David Lewis for emphasizing the importance of this letter.) Geach wrote:

> I've had an idea about de re modalities. Suppose we assume a lot of different possible worlds and speak of the replacement of an individual x in another world W. (Like the different Sexti Tarquinii in Leibniz' fable.) Then x is necessarily-p iff there is no world w containing a replacement of x that is not p: x is possibly-p iff there is some world w containing a replacement of x that is p. (Not every world need contain a replacement of x. The 'replacement' of x in our world is x itself – this is terminological.)

[22] Geach in personal communication with the author (1996).

[23] Smiley in personal communication with the author (2000).

[24] An important influence on Smiley was Leonard (1951). Leonard's statement 'A proposition is said to be *possible* when none of its forms is counter-analytic' (1951: 53) is the idea behind Smiley's translational semantics.

[25] Thanks to David Shoesmith for providing a copy of this lecture handout.

[26] Smiley in personal communication with the author (2000).

[27] Føllesdal even proposes the term 'Kanger–Kripke semantics' for possible worlds semantics (1994: 886). The term is doubly inaccurate.

[28] Lindström (1998) gives a detailed discussion of some of the similarities and differences between Kanger's and Montague's approaches.

[29] Letter quoted with the permission of Kim Kanger.

[30] Thanks to Sten Lindström for providing me with a copy of Kanger (1957c).

[31] Except where otherwise indicated, material in this section is from the author's interviews with Hintikka in 1993 and 2001.

[32] I am grateful to an anonymous referee for first making me aware of the existence of these talks and to Hintikka for describing them.

[33] Binkley in personal communication with the author (1996).

[34] Personal communication (1996).

[35] Translation by Vanessa Scholes.

[36] Drake in personal communication with the author (2000).

[37] Drake (1962) was received in February 1961. The paper formed part of his PhD thesis (Drake, 1963).

[38] Except where otherwise indicated, this section and the next are based on the author's correspondence with Kripke during 1993 and 1996.

[39] Kripke wrote to Prior:

in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like – and for each possible next moment, there are several possibilities for the moment after that. Thus the situation takes the form, not of a linear sequence, but of a 'tree'. (Quoted in Copeland, 1996: 14)

[40] Kripke had corresponded with and met Curry, who was a source of encouragement.

[41] The letter is from Kanger to Richard Routley.

[42] In a review of Kanger's 'The Morning Star Paradox', Hintikka said:

Kanger's discussion of the Morning Star Paradox will in the reviewer's opinion remain a historical landmark as the first philosophical application of an explicit semantical theory of quantified modal logic. (Hintikka, 1969: 306)

[43] See note 38.

[44] Kripke in personal communication with the author (1996).

[45] An important landmark was the publication of Hughes and Cresswell (1968).

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