

POSSIBILITY OF A NEGATIVE ELECTRON GRAVITATIONAL MASS

GENERAL CONSIDERATIONS

The provision of the equivalence of inertial and gravitational mass by the Mills theory of fundamental particles wherein spacetime is Riemannian due to its relativistic correction with particle production permits the correct derivation of the General Theory. In the case of ordinary matter (an example of an extraordinary state of matter called a hyperbolic electron is given *infra*), the nature of chemical bonding is electric and magnetic, and the angular momentum of each bound electron is always $\hbar$ independent of material such as wood or metal. The angular momentum with a central field is given by Eq. (1.57). In this case, each infinitesimal point of the orbitsphere of mass m_i is the inertial mass according to the inertial angular momentum. It also is the gravitational mass according to the gravitational angular momentum. The inertial and gravitational mass of electrons and nucleons in ordinary matter are equivalent.

The provision of the two-dimensional nature of matter permits the unification of atomic, subatomic, and cosmological gravitation. The unified theory of gravitation is derived by first establishing a metric. A space in which the curvature tensor has the following form:

$$R_{\mu\nu,\alpha\beta} = K\ (g_{\nu\alpha} g_{\mu\beta} - g_{\mu\alpha} g_{\nu\beta}) \tag{26.1}$$

is called a space of constant curvature; it is a four-dimensional generalization of Friedmann-Lobachevsky space. The constant K is called the constant of curvature. *The curvature of spacetime results from a discontinuity of matter having curvature confined to two spatial dimensions. This is the property of all matter as an orbitsphere.* Consider an isolated orbitsphere and radial distances, r, from its center. *For r less than r_n there is no mass; thus, spacetime is flat or Euclidean.* The curvature tensor applies to all space of the inertial frame considered; thus, for r less than r_n, $K = 0$. At $r = r_n$ there exists a discontinuity of mass of the orbitsphere. This results in a discontinuity of the curvature tensor for radial distances greater than or equal to r_n. The discontinuity requires relativistic corrections to spacetime itself. It requires radial length contraction and time dilation that results in the curvature of spacetime. The gravitational radius of the orbitsphere and infinitesimal temporal displacement in spacetime which is curved by the presence of the orbitsphere are derived in the Gravity Section.

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity. The correction is based on the boundary conditions that no signal can travel faster that the speed of light including the gravitational field that propagates following particle production from a photon wherein the particle has a finite gravitational velocity given by Newton's Law of Gravitation. The separation of proper time between two events x^μ and $x^\mu + dx^\mu$ given by Eq. (23.38), the Schwarzschild metric [1-2], is

$$d\tau^2 = \left(1 - \frac{2Gm_0}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2Gm_0}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.2}$$

Eq. (26.2) can be reduced to Newton's Law of Gravitation for r_g, the gravitational radius of the particle, much less than r_α^*, the radius of the particle at production ($\frac{r_g}{r_\alpha^*} << 1$), where the radius of the particle is its Compton wavelength bar ($r_\alpha^* = \lambda\!\!\!^{-}_c$).

$$F = \frac{Gm_1 m_2}{r^2} \tag{26.3}$$

where G is the Newtonian gravitational constant. Eq. (26.2) relativistically corrects Newton's gravitational theory. In an analogous manner, Lorentz transformations correct Newton's laws of mechanics.

The effects of gravity preclude the existence of inertial frames in a large region, and only local inertial frames, between which relationships are determined by gravity are possible. In short, the effects of gravity are only in the determination of the local inertial frames. The frames depend on gravity, and the frames describe the spacetime background of the motion of matter. Therefore, differing from other kinds of forces, gravity which influences the motion of matter by determining the properties of spacetime is itself described by the metric of spacetime. It was demonstrated in the Gravity Section that gravity arises from the two spatial dimensional mass density functions of the fundamental particles.

It is demonstrated in the One Electron Atom Section that a bound electron is a two-dimensional spherical shell— an orbitsphere. On the atomic scale, the curvature, K, is given by $\frac{1}{r_n^2}$, where r_n is the radius of the radial delta function of the orbitsphere. The velocity of the electron is a constant on this two dimensional sphere. It is this local, positive curvature of the electron that causes gravity. It is worth noting that all ordinary matter, comprised of leptons and quarks, has positive curvature. Euclidean plane geometry asserts that (in a plane) the sum of the angles of a triangle equals 180°. In fact, this is the definition of a flat surface. For a triangle on an orbitsphere the sum of the angles is greater than 180°, and the orbitsphere has *positive curvature*. For some surfaces the sum of the angles of a triangle is less than 180°; these are said to have *negative curvature.*

sum of angles of a triangle	type of surface
> 180°	positive curvature
= 180°	flat
< 180°	negative curvature

The measure of Gaussian curvature, K, at a point on a two dimensional surface is

$$K = \frac{1}{r_1 r_2} \tag{26.4}$$

the inverse product of the radius of the maximum and minimum circles, r_1 and r_2, which fit the surface at the point, and the radii are normal to the surface at the point. By a theorem of Euler, these two circles lie in orthogonal planes. For a sphere, the radii of the two circles of curvature are the same at every point and equivalent to the radius of a great circle of the sphere. Thus, the sphere is a surface of constant curvature;

$$K = \frac{1}{r^2} \tag{26.5}$$

at every point. In case of positive curvature of which the sphere is an example, the circles fall on the same side of the surface, but when the circles are on opposite sides, the curve has negative curvature. A saddle, a cantenoid, and a pseudosphere are negatively curved. The general equation of a saddle is

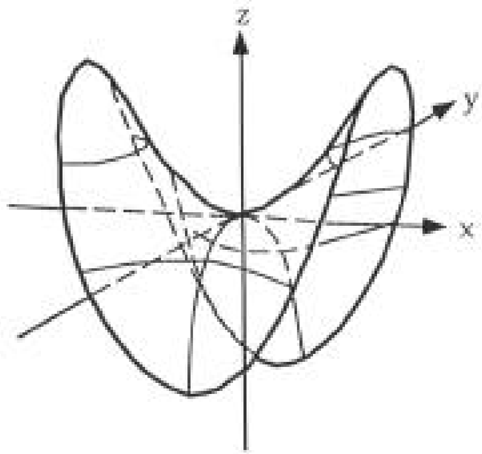
$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{26.6}$$

where a and b are constants. The curvature of the surface of Eq. (26.6) is

$$K = \frac{-1}{4a^2b^2}\left[\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{1}{4}\right]^{-2} \tag{26.7}$$

A saddle is shown schematically in Figure 26.1.

Figure 26.1. A saddle.

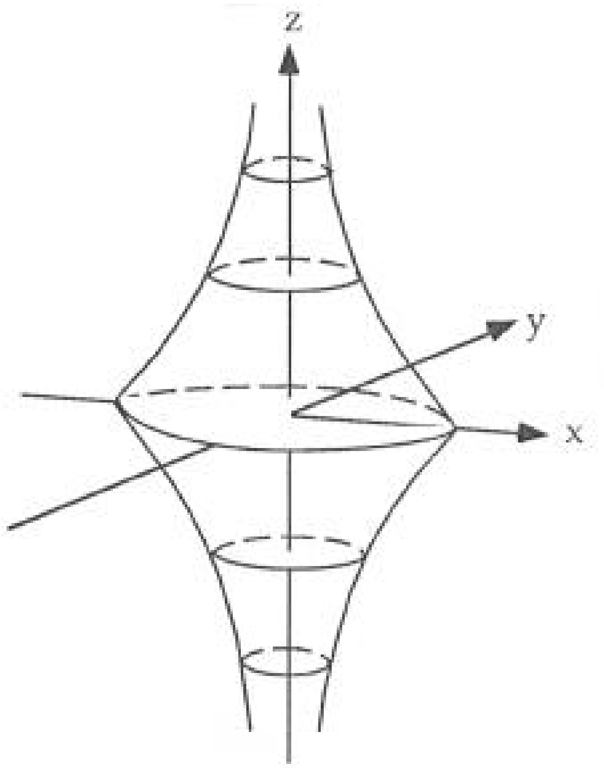


A pseudosphere is constructed by revolving the tractrix about its asymptote. For the tractrix, the length of any tangent measured from the point of tangency to the x-axis is equal to the height R of the curve from its asymptote-in this case the x-axis. The pseudosphere is a surface of constant negative curvature. The curvature, K

$$K = \frac{-1}{r_1 r_2} = \frac{-1}{R^2} \qquad (26.8)$$

given by the product of the two principal curvatures on opposite sides of the surface is equal to the inverse of R squared at every point where R is the equitangent. R is also known as the radius of the pseudosphere. A pseudosphere is shown schematically in Figure 26.2.

Figure 26.2. A pseudosphere.



In the case of a sphere, surfaces of constant potential are concentric spherical shells. The general law of potential for surfaces of constant curvature is

$$V = \frac{1}{4\pi\varepsilon_o}\sqrt{\frac{1}{r_1 r_2}} = \frac{1}{4\pi\varepsilon_o R} \tag{26.9}$$

In the case of a pseudosphere the radii r_1 and r_2, the two principal curvatures, represent the distances measured along the normal from the negative potential surface to the two sheets of its evolute, envelop of normals (cantenoid and x-axis). The force is given as the gradient of the potential which is proportional to $\frac{1}{r^2}$ in the case of a sphere.

All matter is comprised of fundamental particles, and all fundamental particles exists as mass confined to two spatial dimensions. The particle's velocity surface is positively curved in the case of an orbitsphere, or the velocity surface is negatively curved in the case of an electron as a hyperboloid (hereafter called a hyperbolic electron given in the Hyperbolic Electrons Section). The effect of this "local" curvature on the non-local spacetime is to cause it to be Riemannian, in the case of an orbitsphere, or hyperbolic, in the case of a hyperbolic electron, as opposed to Euclidean which is manifest as a gravitational field or an antigravitational field, respectively. Thus, the spacetime is curved with constant spherical curvature in the case of an orbitsphere, or spacetime is curved with hyperbolic curvature in the case of a hyperbolic electron.

The relativistic correction for spacetime dilation and contraction due to the production of a particle with positive curvature is given by Eq. (23.17)

$$f(r) = \left(1 - \left(\frac{v_g}{c}\right)^2\right) \tag{26.10}$$

The derivation of the relativistic correction factor of spacetime was based on the constant maximum velocity of light and a finite positive Newtonian gravitational velocity v_g of the particle given by

$$v_g = \sqrt{\frac{2Gm_0}{r}} = \sqrt{\frac{2Gm_0}{\bar{\lambda}_C}} \tag{26.11}$$

Consider a Newtonian gravitational radius, r_g, of each orbitsphere of the particle production event, each of mass m

$$r_g = \frac{2Gm}{c^2} \tag{26.12}$$

where G is the Newtonian gravitational constant. Substitution of Eq. (26.11) or Eq. (26.12) into the Schwarzschild metric Eq. (26.2), gives

$$d\tau^2 = \left(1 - \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.13}$$

and

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.14}$$

respectively. The solutions for the Schwarzschild metric exist wherein the relativistic correction to the gravitational velocity v_g and the gravitational radius r_g are of the opposite sign (i.e. negative). In these cases the Schwarzschild metric Eq. (26.2), is

$$d\tau^2 = \left(1 + \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 + \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.15}$$

and

$$d\tau^2 = \left(1 + \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 + \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.16}$$

The metric given by Eqs. (26.13-26.14) corresponds to positive curvature. The metric given by Eqs. (26.15-26.16) corresponds to negative curvature. The negative solution arises naturally as a match to the boundary condition of matter with a velocity function having negative curvature. Consider the case of pair production given in the Gravity Section. The photon equation given in the Equation of the

Photon Section is equivalent to the electron and positron functions given by in the One Electron Atom Section. The velocity of any point on the positively curved electron orbitsphere is constant which correspond to the trigonometric function given in Eqs. (1.68-1.69). At particle production, the relativistic corrections to spacetime due to the constant gravitational velocity v_g are given by Eqs. (26.13-26.14). In the case of negative curvature, the electron velocity as a function of position is not constant. It may be described by a harmonic variation which corresponds to an imaginary velocity. The trigonometric function of the positively curved electron orbitsphere given in Eqs. (1.68-1.69) becomes a hyperbolic function (e.g. $\cosh$) in the case of a negatively curved electron. Substitution of an imaginary velocity with respect to a gravitating body into Eq. (26.13) givens Eq. (26.15). Substitution a negative radius of curvature with respect to a gravitating body into Eq. (26.14) gives Eq. (26.16). Thus, antigravity can be created by forcing matter into negative curvature of the velocity surface. A fundamental particle with negative curvature of the velocity surface would experience a central but repulsive force with a gravitating body comprised of matter of positive curvature of the velocity surface.

POSITIVE, ZERO, AND NEGATIVE GRAVITATIONAL MASS

In the case of Einstein's gravity equation (Eq. (23.40)), the Einstein's Tensor $G_{\mu\nu}$, is equal to the stress-energy-momentum tensor $T_{\mu\nu}$. The only possibility is for the gravitational mass to be equivalent to the inertial mass. A particle of zero or negative gravitational mass is not possible. However, it is shown in the Gravity Section that the correct basis of gravitation is not according to Einstein's equation (Eq. (23.40)); instead the origin of gravity is the relativistic correction of spacetime itself which is analogous to the special relativistic corrections of inertial parameters-- increase in mass, dilation in time, and contraction in length in the direction of constant relative motion of separate inertial frames. On this basis, the observed acceleration of the cosmos is predict as given in the Cosmology Section.

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity. Matter arises during particle production from a photon. According to Newton's Law of Gravitation, the production of a particle of finite mass gives rise to a gravitational velocity of the particle. The gravitational velocity determines the energy and the corresponding eccentricity and trajectory of the gravitational orbit of the particle. The eccentricity e given by Newton's differential equations of motion in the case of the central field

(Eq. (23.49-23.50)) permits the classification of the orbits according to the total energy E as follows [3]:

$E<0$,	$e<1$	ellipse
$E<0$,	$e=0$	circle (special case of ellipse)
$E=0$,	$e=1$	parabolic orbit
$E>0$,	$e>1$	hyperbolic orbit

(26.17)

Since $E=T+V$ and is constant, the closed orbits are those for which $T<|V|$, and the open orbits are those for which $T\geq|V|$. It can be shown that the time average of the kinetic energy, $<T>$, for elliptic motion in an inverse square field is $1/2$ that of the time average of the potential energy, $<V>$. $<T>=1/2<V>$.

In the case that a particle of inertial mass m is observed to have a speed v_0, a distance from a massive object r_0, and a direction of motion makes that an angle ϕ with the radius vector from the object (including a particle) of mass M, the total energy is given by

$$E=\frac{1}{2}mv^2-\frac{GMm}{r}=\frac{1}{2}mv_0^2-\frac{GMm}{r_0}=\text{constant} \tag{26.18}$$

The orbit will be elliptic, parabolic, or hyperbolic, according to whether E is negative, zero, or positive. Accordingly, if v_0^2 is less than, equal to, or greater than $\frac{2GM}{r_0}$, the orbit will be an ellipse, a parabola, or a hyperbola, respectively. Since h, the angular momentum per unit mass, is

$$h=L/m=|\mathbf{r}\times\mathbf{v}|=r_0v_0\sin\phi \tag{26.19}$$

The eccentricity e, from Eq. (23.63) may be written as

$$e=[1+\left(v_0^2-\frac{2GM}{r_0}\right)\frac{r_0^2v_0^2\sin^2\phi}{G^2M^2}]^{1/2} \tag{26.20}$$

As shown in the Gravity Section (Eq. (23.35)), the production of a particle requires that the velocity of each of the point masses of the particle is equivalent to the Newtonian gravitational escape velocity v_g of the superposition of the point masses of the antiparticle.

$$v_g=\sqrt{\frac{2Gm}{r}}=\sqrt{\frac{2Gm_0}{\bar{\lambda}_C}} \tag{26.21}$$

From Eq. (26.20) and Eq. (26.17), the eccentricity is one and the particle production trajectory is a parabola relative to the center of mass of the antiparticle. The right-hand side of Eq. (23.43) represents the correction to the laboratory coordinate metric for time corresponding to the relativistic correction of spacetime by the particle production event. Riemannian space is conservative. Only changes in the metric of spacetime during particle production must be considered. The changes must be conservative. For example, pair production occurs in the presence of a heavy body. A nucleus which existed before the production event only serves to conserve momentum but is not a factor in determining the change in the properties of spacetime as a consequence of the pair production event. The effect of this and other external gravitating bodies are equal on the photon and resulting particle and antiparticle and do not effect the boundary conditions for particle production. For particle production to occur, the particle must possess the escape velocity relative to the antiparticle where Eqs. (23.34), (23.48), and (23.140) apply. In other cases not involving particle production such as a special electron scattering event wherein hyperbolic electron production occurs as given *infra,* the presence of an external gravitating body must be considered. The curvature of spacetime due to the presence of a gravitating body and the constant maximum velocity of the speed of light comprise boundary conditions for hyperbolic electron production from a free electron.

With particle production, the form of the outgoing gravitational field front traveling at the speed of light (Eq. (23.10)) is

$$f\left(t-\frac{r}{c}\right) \tag{26.22}$$

At production, the particle must have a finite velocity called the gravitational velocity according to Newton's Law of Gravitation. In order that the velocity does not exceed c in any frame including that of the particle having a finite gravitational velocity, the laboratory frame of an incident photon that gives rise to the particle, and that of a gravitational field propagating outward at the speed of light, spacetime must undergo time dilation and length contraction due to the production event. During particle production the speed of light as a constant maximum as well as phase matching and continuity conditions require the following form of the squared displacements due to constant motion along two orthogonal axes in polar coordinates:

$$(c\tau)^2+(v_gt)^2=(ct)^2 \tag{26.23}$$

$$(c\tau)^2=(ct)^2-(v_gt)^2 \tag{26.24}$$

$$\tau^2=t^2\left(1-\left(\frac{v_g}{c}\right)^2\right) \tag{26.25}$$

Thus,

$$f(r) = \left(1 - \left(\frac{v_g}{c}\right)^2\right) \tag{26.26}$$

(The derivation and result of spacetime time dilation is analogous to the derivation and result of special relativistic time dilation given by Eqs. (22.11-22.15).) Consider a gravitational radius, r_g, of each orbitsphere of the particle production event, each of mass m

$$r_g = \frac{2Gm}{c^2} \tag{26.27}$$

where G is the Newtonian gravitational constant. Substitution of Eq. (26.11) or Eq. (26.12) into the Schwarzschild metric Eq. (26.2), gives the general form of the metric due to the relativistic effect on spacetime due to mass m_0.

$$d\tau^2 = \left(1 - \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.28}$$

and

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.29}$$

respectively. Masses and their effects on spacetime superimpose; thus, the metric corresponding to the Earth is given by substitution of the mass of the Earth M for m in Eqs. (26.13-26.14). The corresponding Schwarzschild metric Eq. (26.2) is

$$d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.30}$$

Gravitational and electromagnetic forces are both inverse squared central forces. The inertial mass corresponds to the inertial angular momentum and the gravitational mass corresponds to the gravitational angular momentum. In the case that an electron is bound in by electromagnetic forces in a nonradiative orbit, the following condition from the particle production relationships given by Eq. (24.41) hold

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2 \lambda\!\!\!^{-}_C}}}{\alpha} = i\frac{v_g}{\alpha c} \tag{26.31}$$

The gravitational and inertial angular momentum correspond to the same mass; thus, the inertial and gravitational masses are identically equal for all matter in a stable bound state.

Consider the case that the radius in Eq. (26.30) goes to infinity. From Eq. (26.20) and Eq. (26.17) in the case that r_0 goes to infinity, the eccentricity is always greater than or equal to one and approaches infinity, and the trajectory is a parabola or a hyperbola. The gravitational velocity (Eq. (26.21)) where $m = M$ goes to zero. This condition must hold from all r_0; thus, the free electron is not effected by the gravitational field of a massive object, but has inertial mass determined by the conservation of the angular momentum of $\hbar$ as shown by Eqs. (3.14-3.15). From the Electron in Free Space Section, the free electron has a velocity distribution given by

$$\mathbf{v}(\rho,\phi,z,t) = \left[\pi\left(\frac{\rho}{2\rho_o}\right)\frac{\hbar}{m_e\sqrt{\rho_0^{\,2}-\rho^2}}\,\mathbf{i}_\phi\right]$$
$$\mathbf{v}(\rho,\phi,z,t) = \left[\pi\left(\frac{\rho}{2\rho_o}\right)\frac{\hbar}{m_e\rho_o\sqrt{1-\left(\frac{\rho}{\rho_o}\right)^2}}\,\mathbf{i}_\phi\right] \quad (26.32)$$

The velocity function is a paraboloid in a two dimensional plane. The corresponding gravity field front corresponds to a radius at infinity in Eq. (26.22). As a consequence, an ionized or free electron has a gravitational mass that is zero; whereas, the inertial mass is constant (e.g. equivalent to its mass energy given by Eq. (24.13)). Minkowski space applies to the free electron.

In the Electron in Free Space Section, a free electron is shown to be a two-dimensional plane wave—a flat surface. Because the gravitational mass depends on the positive curvature of a particle, a free electron has inertial mass but not gravitational mass. The experimental mass of the free electron measured by Witteborn [4] using a free fall technique is less than 0.09 m_e, where m_e is the inertial mass of the free electron $\left(9.109534\ X\ 10^{-31}\ kg\right)$. Thus, ***a free electron is not gravitationally attracted to ordinary matter, and the gravitational and inertial masses are not equivalent***. Furthermore, it is possible to give the electron velocity function negative curvature and, therefore, cause antigravity.

As is the case of special relativity, the velocity of a particle in the presence of a gravitating body is relative. In the case that the relative gravitational velocity is imaginary, the eccentricity is always greater than one, and the trajectory is a hyperbola. This case corresponds to a hyperbolic electron wherein gravitational mass is effectively negative and the inertial mass is constant (e.g. equivalent to its mass energy given by Eq. (24.13)). The formation of a hyperbolic electron occurs over the time that the plane wave free electron scatters from the neutral atom. Huygens' principle, Newton's law of Gravitation, and the constant speed of light in all inertial frames provide the boundary conditions to determine the metric corresponding to the hyperbolic electron. From Eq. (26.71), the velocity $\mathbf{v}(\rho,\phi,z,t)$ on a two dimensional sphere in spherical coordinates is

$$\mathbf{v}(r,\theta,\phi,t) = \left[\frac{\hbar}{m_e r_0 \sin\theta}\delta\left(r - r_0\right)\mathbf{i}_\phi\right] \tag{26.33}$$

With hyperbolic electron production, the form of the outgoing gravitational field front traveling at the speed of light (Eq. (23.10)) is

$$f\left(t - \frac{r}{c}\right) \tag{26.34}$$

During hyperbolic electron production the speed of light as a constant maximum as well as phase matching and continuity conditions require the following form of the squared displacements due to constant motion along two orthogonal axes in polar coordinates:

$$(c\tau)^2 + \left(v_g t\right)^2 = (ct)^2 \tag{26.35}$$

According to Eq. (3.11), the velocity of the electron on the two dimension sphere approaches the speed of light at the angular extremes ($\theta = 0$ and $\theta = \pi$), and the velocity is harmonic as a function of theta. The speed of any signal can not exceed the speed of light. Therefore, the outgoing two dimensional spherical gravitational field front traveling at the speed of light and the velocity of the electron at the angular extremes require that the relative gravitational velocity must be radially outward. The relative gravitational velocity squared of the term $\left(v_g t\right)^2$ of Eq. (26.35) must be negative. In this case, the relative gravitational velocity may be considered imaginary which is consistent with the velocity as a harmonic function of theta. The energy of the orbit of the hyperbolic electron must always be greater than zero which corresponds to a hyperbolic trajectory and an eccentricity greater than one (Eq. (26.17) and Eq. (26.20)). From Eq. (26.20) and Eq. (26.21) with the requirements that the relative gravitational velocity must be imaginary and the energy of the orbit must always be positive, the relative gravitational velocity for a hyperbolic electron produced in the presence of the gravitational field of the Earth is

$$v_g = i\sqrt{\frac{2GM}{r}} \tag{26.36}$$

where M is the mass of the Earth. Substitution of Eq. (26.36) into Eq.

(26.35) gives

$$(c\tau)^2 = (ct)^2 + (v_g t)^2 \tag{26.37}$$

$$\tau^2 = t^2\left(1 + \left(\frac{v_g}{c}\right)^2\right) \tag{26.38}$$

Thus,

$$f(r) = \left(1 + \left(\frac{v_g}{c}\right)^2\right) \tag{26.39}$$

Consider a gravitational radius, r_g, of a massive object of mass M relative to a hyperbolic electron at the production event that is negative to match the boundary condition of a negatively curved velocity surface

$$r_g = -\frac{2GM}{c^2} \tag{26.40}$$

where G is the Newtonian gravitational constant. Substitution of Eq. (26.36) or Eq. (26.40) into the Schwarzschild metric Eq. (26.2), gives the general form of the metric due to the relativistic effect on spacetime due to a massive object of mass M relative to the hyperbolic electron.

$$d\tau^2 = \left(1 + \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 + \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.41}$$

and

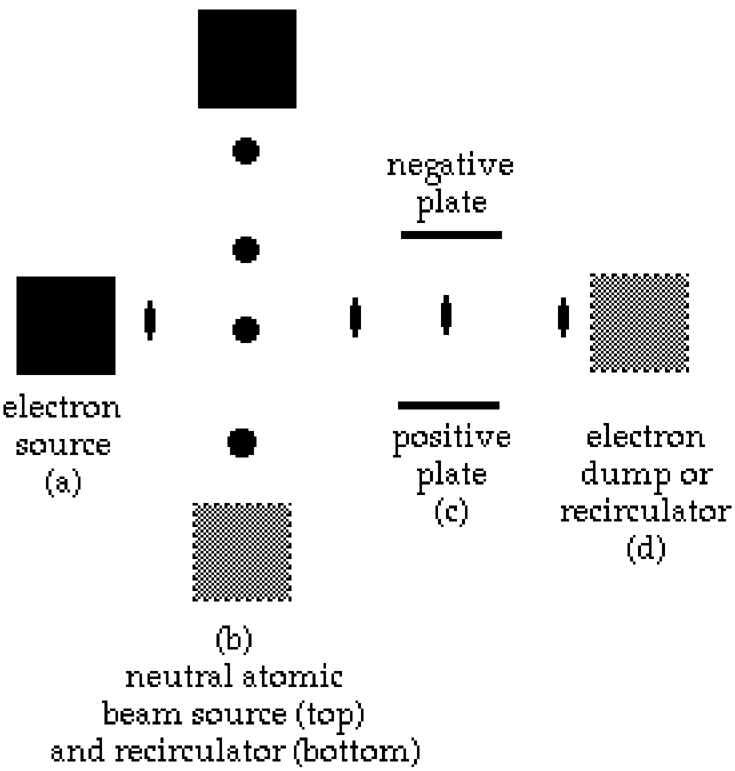
$$d\tau^2 = \left(1 + \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 + \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{26.42}$$

respectively.

ANTIGRAVITY DEVICE

It is possible to give the velocity function of electrons negative curvature by elastically scattering electrons of an electron beam from atoms such that electrons with negatively curved velocity surfaces (hyperbolic electrons) emerge. The emerging beam of electrons with negatively curved velocity surfaces experience an antigravitational force (on the Earth), and the beam will tend to move upward (away from the Earth). To use this invention for propulsion or levitation, the antigravitational force of the electron beam must be transferred to a negatively charged plate. The Coulombic repulsion between the beam of electrons and the negatively charged plate will cause the plate (and anything connected to the plate) to lift. Figure 26.3 gives a schematic of an antigravity levitation device.

Figure 26.3. An antigravity device.



(a) a beam of electrons is generated and directed to the neutral atomic beam

(b) scattering of the electrons of the electron beam by the neutral atom beam gives the electrons negative curvature of their velocity surfaces, and the electrons experience an antigravitational force upward (away from the earth)

(c) the electrons, which would normally bend down toward the positive plate but do not because of the antigravitational force, repel the negative plate and attract the positive plate, and transfer the antigravity force to the object to be lifted or propelled

(d) the electrons are collected or recirculated back to the electron beam

HYPERBOLIC ELECTRONS

A method and means to produce an antigravitational force for propulsion and/or levitation comprises a source of fundamental particles including electrons and a source of neutral atoms. The source of electrons produces a free electron beam, and the source of neutral atoms produces a free atom beam. The two beams intersect such that the neutral atoms cause elastic incompressible scattering of the electrons of the electron beam to form hyperbolic electrons. In a preferred embodiment, the de Broglie wavelength of each electron is given by

$$\lambda_o = \frac{h}{m_e v_z} = 2\pi\rho_o \tag{26.43}$$

where ρ_o is the radius of the free electron in the xy-plane, the plane perpendicular to its direction of propagation. The velocity of each electron follows from Eq. (26.43)

$$v_z = \frac{h}{m_e \lambda_o} = \frac{h}{m_e 2\pi\rho_o} = \frac{\hbar}{m_e \rho_o} \tag{26.44}$$

The elastic electron scattering in the far field is given by the Fourier Transform of the aperture function as described in Electron Scattering by Helium Section. The convolution of a uniform plane wave with on orbitsphere of radius z_o is given by Eq. (8.43) and Eq. (8.44).

The aperture distribution function, $a(\rho,\phi,z)$, for the scattering of an incident plane wave by the He atom is given by the convolution of the plane wave function with the two electron orbitsphere Dirac delta function of $radius = 0.567a_o$ and charge/mass density of $\frac{2}{4\pi(0.567a_o)^2}$. For radial units in terms of a_o

$$a(\rho,\phi,z) = \pi(z) \quad \frac{2}{4\pi(0.567a_o)^2}[\delta(r - 0.567a_o)] \tag{26.45}$$

where $a(\rho,\phi,z)$ is given in cylindrical coordinates, $\pi(z)$, the xy-plane wave is given in Cartesian coordinates with the propagation direction along the z-axis, and the He atom orbitsphere function, $\frac{2}{4\pi(0.567a_o)^2}[\delta(r - 0.567a_o)]$, is given in spherical coordinates.

$$a(\rho,\phi,z) = \frac{2}{4\pi(0.567a_o)^2}\sqrt{(0.567a_o)^2 - z^2}\,\delta\left(r - \sqrt{(0.567a_o)^2 - z^2}\right) \tag{26.46}$$

The convolution of the charge-density equation of a free electron given by Eq. (3.7) with an orbitsphere of radius z_o follows from Eq. (3.7) and Eq. (26.46)

$$\rho_m(\rho,\phi,z) = \sqrt{{\rho_0}^2 - \rho^2}\sqrt{{z_0}^2 - z^2}\,\delta\left(\rho - \sqrt{{z_0}^2 - z^2}\right) \tag{26.47}$$

Substitution of Eq. (26.47) into Eq. (8.45) gives

$$F(s) = \frac{1}{{z_o}^2}\int_{-z_o}^{z_o}\sqrt{{\rho_o}^2 - ({z_o}^2 - z^2)}\,({z_o}^2 - z^2)\,J_o\left(s\sqrt{{z_o}^2 - z^2}\right)e^{iwz}\,dz \tag{26.48}$$

Substitution $\frac{z}{z_o} = -\cos\theta$ into Eq. (26.48) gives

$$F(s) = \int_0^{\pi} \sqrt{\rho_o^2 - z_o^2 \sin^2\theta}\, \sin^3\theta J_o(sz_o \sin\theta) e^{iz_0 w \cos\theta} d\theta \tag{26.49}$$

when $\rho_o = z_o$, Eq. (26.49) becomes

$$F(s) = z_o \int_0^{\pi} \cos\theta \sin^3\theta J_o(sz_o \sin\theta) e^{iz_0 w \cos\theta} d\theta \tag{26.50}$$

The function of the scattered electron in the far field is given by the Fourier Transform integral, Eq. (26.50). Eq. (26.50) is equivalent to the Fourier Transform integral of $\cos\theta$ times the Fourier Transform integral given by of Eq. (8.47) with the latter result given by Eq. (8.50).

$$F(s) = \left[\frac{2\pi}{(z_o w)^2 + (z_o s)^2}\right]^{\frac{1}{2}}$$

$$\left\{ 2\left[\frac{z_o s}{(z_o w)^2 + (z_o s)^2}\right] J_{3/2}\left[((z_o w)^2 + (z_o s)^2)^{1/2}\right] - \left[\frac{z_o s}{(z_o w)^2 + (z_o s)^2}\right]^2 J_{5/2}\left[((z_o w)^2 + (z_o s)^2)^{1/2}\right] \right\} \tag{26.51}$$

where

$$s = \frac{4\pi}{\lambda}\sin\frac{\theta}{2};\ w = 0\ (units\ of\ \mathring{A}^{-1}) \tag{26.52}$$

A very important theorem of Fourier analysis states that the Fourier Transform of a product is the convolution of the individual Fourier Transforms. The Fourier Transform of $\cos\theta$ is

$$\frac{[\delta(\Theta - \Theta_o) + \delta(\Theta + \Theta_o)]}{2} \tag{26.53}$$

The Fourier Transform integral, Eq. (26.50), is the convolution of Eqs. (26.51-26.52) and Eq. (26.53). The convolution gives the result that Eq. (26.52) is given by

$$s = \frac{4\pi}{\lambda}\sin\left(\frac{\theta - \Theta_o}{2}\right);\ w = 0\ (units\ of\ \mathring{A}^{-1}) \tag{26.54}$$

Given that $z = z_o\cos\theta$, the mass density function of each electron having a de Broglie wavelength λ_o given by Eq. (26.43) corresponding to λ in Eq. (26.54) which is elastically scattered by an atom having a radius of $z_o = \rho_o$ is given by Eqs. (26.51) and (26.54). The replacement of Π(z), the xy-plane wave corresponding to the superposition of many electrons scattered from an atomic beam with the function of a single electron propagating in the z-direction (Eq. (3.7)) gives rise to the ***electron density function on a two dimensional sphere*** of

$$\rho_m(\rho,\phi,z) = Nm_e\sqrt{\rho_0^2 - z^2}\,\delta\left(\rho - \sqrt{\rho_0^2 - z^2}\right) \tag{26.55}$$

centered at a scattering angle of Θ_o. With the condition $z_o = \rho_o$, the elastic electron scattering angle in the far field Θ_o is determined by the boundary conditions of the curvature of spacetime due to the presence of a gravitating body and the constant maximum velocity of the speed of light. The far field condition must be satisfied with respect to electron scattering and the gravitational orbital equation. The former condition is met by Eq. (26.51) and Eq. (26.54). The latter is derived in the Preferred Embodiment of an Antigravity Device Section and is met by Eq. (26.103) where the far field angle of the hyperbolic gravitational trajectory ϕ is equivalent to Θ_o.

The electron mass/charge density function, $\rho_m(\rho,\phi,z)$, is given in cylindrical coordinates, and N is the normalization factor. The charge density, mass density, velocity, current density, and angular momentum functions are derived in the same manner as for the free electron given in the Electron in Free Space Section except that the scattered electron is symmetric about the z-axis. The total mass is m_e. Thus, Eq. (26.55) must be normalized.

$$m_e = N \int_{-\rho_0}^{\rho_0} \int_0^{2\pi} \int_{-\infty}^{\infty} \sqrt{{\rho_0}^2 - z^2}\,\delta\left(\rho - \sqrt{{\rho_0}^2 - z^2}\right) \rho d\rho\, d\phi dz \tag{26.56}$$

$$N = \frac{m_e}{\frac{8}{3}\pi {\rho_0}^3} \tag{26.57}$$

The mass density function, $\rho_m(\rho,\phi,z)$, of the scattered electron is

$$\rho_m(\rho,\phi,z) = \frac{m_e}{\frac{8}{3}\pi {\rho_0}^3} \sqrt{{\rho_0}^2 - z^2}\,\delta\left(\rho - \sqrt{{\rho_0}^2 - z^2}\right)$$
$$\rho_m(\rho,\phi,z) = \frac{m_e}{\frac{8}{3}\pi {\rho_0}^3} \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}\,\delta\left(\rho - \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}\right) \tag{26.58}$$

and charge-density distribution, $\rho_e(\rho,\phi,z)$, is

$$\rho_e(\rho,\phi,z) = \frac{e}{\frac{8}{3}\pi {\rho_0}^3} \sqrt{{\rho_0}^2 - z^2}\,\delta\left(\rho - \sqrt{{\rho_0}^2 - z^2}\right)$$
$$\rho_e(\rho,\phi,z) = \frac{e}{\frac{8}{3}\pi {\rho_0}^3} \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}\,\delta\left(\rho - \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}\right) \tag{26.59}$$

The magnitude of the angular velocity of the helium orbitsphere is given by Eq. (1.55) is

$$\omega = \frac{\hbar}{m_e r^2} \tag{26.60}$$

where $r = r_0 = \rho_0 = z_0 = 0.567 a_0$ and a_0 is the Bohr radius. The current-density function of the scattered electron, $\mathbf{K}(\rho,\phi,z,t)$, is the projection along the z-axis of the integral of the product of the projections of the charge of the orbitsphere (Eq. (3.3)) times the angular velocity as a function of the radius r of an ionizing orbitsphere (Eq. (3.9)) for $r = r_o$ to $r = \infty$. The integral is

$$\int_{r_o}^{\infty} \omega \boldsymbol{\pi}(z) \quad \delta(r - r_o)dr = \frac{e}{\frac{8}{3}\pi r_0^{\;3}} \int_{r_o}^{\infty} \frac{\hbar}{m_e r^2} \sqrt{r_0^2 - z^2}\,\delta\left(r - \sqrt{r_0^2 - z^2}\right)dr \tag{26.61}$$

The projection of Eq. (26.61) along the z-axis is

$$\mathbf{J}(\rho,\phi,z,t) = \left[\frac{e}{\frac{8}{3}\pi \rho_0^{\;3}} \frac{\hbar}{m_e \sqrt{\rho_0^{\;2} - z^2}} \delta\left(\rho - \sqrt{\rho_0^2 - z^2}\right) \mathbf{i}_\phi \right] \tag{26.62}$$

The velocity $\mathbf{v}(\rho,\phi,z,t)$ along the z-axis is

$$\mathbf{v}(\rho,\phi,z,t) = \left[\frac{\hbar}{m_e \sqrt{\rho_0^{\;2} - z^2}} \delta\left(\rho - \sqrt{\rho_0^2 - z^2}\right) \mathbf{i}_\phi \right]$$

$$\mathbf{v}(\rho,\phi,z,t) = \left[\frac{\hbar}{m_e \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}} \delta\left(\rho - \rho_0 \sqrt{1 - \left(\frac{z}{\rho_0}\right)^2}\right) \mathbf{i}_\phi \right] \tag{26.63}$$

where $\rho_o = r_o$. The angular momentum, $\mathbf{L}$, is given by

$$\mathbf{L}\mathbf{i}_z = m_e r^2 \omega = \mathbf{L} = mr^2 \mathbf{w} = m\mathbf{r} \times \mathbf{v} \tag{26.64}$$

Substitution of m_e for e in Eq. (26.62) followed by substitution into Eq. (26.64) gives the angular momentum density function, $\mathbf{L}$

$$\mathbf{L}\mathbf{i}_z = \frac{m_e}{\frac{8}{3}\pi \rho_0^{\;3}} \frac{\hbar}{m_e \sqrt{\rho_0^{\;2} - z^2}} \rho^2 \delta\left(\rho - \sqrt{\rho_0^2 - z^2}\right) \tag{26.65}$$

The total angular momentum of the scattered electron is given by integration over the two dimensional negatively curved surface having the angular momentum density given by Eq. (26.65).

$$\mathbf{L}\mathbf{i}_z = \int_{-\rho_0}^{\rho_0} \int_0^{2\pi} \int_{-\infty}^{\infty} \frac{m_e}{\frac{8}{3}\pi \rho_0^{\;3}} \frac{\hbar}{m_e \sqrt{\rho_0^{\;2} - z^2}} \delta\left(\rho - \sqrt{\rho_0^2 - z^2}\right) \rho^2 \rho\, d\rho\, d\phi\, dz \tag{26.66}$$

$$\mathbf{L}\mathbf{i}_z = \hbar \tag{26.67}$$

Eq. (26.67) is in agreement with Eq. (1.130); thus, the scalar sum of the magnitude of the angular momentum is conserved.

The mass, charge, and current of the scattered electron exist on a two dimension sphere which may be given in spherical coordinates where theta is with respect to the z-axis of the original cylindrical coordinate system. The mass density function, $\rho_m(r,\theta,\phi)$, of the scattered electron in spherical coordinates is

$$\rho_m(r,\theta,\phi) = \frac{m_e}{\frac{8}{3}\pi r_0^3} r_0 \sin^2\theta\,\delta(r - r_0) \tag{26.68}$$

The charge-density distribution, $\rho_e(r,\theta,\phi)$, in spherical coordinates is

$$\rho_e(r,\theta,\phi) = \frac{e}{\frac{8}{3}\pi r_0^3} r_0 \sin^2\theta\,\delta(r - r_0) \tag{26.69}$$

The current density function $\mathbf{J}(r,\theta,\phi,t)$, in spherical coordinates is

$$\mathbf{J}(r,\theta,\phi,t) = \left[\frac{e}{\frac{8}{3}\pi r_0^2}\frac{\hbar}{m_e r_0^2}\sin\theta\,\delta(r - r_0)\,\mathbf{i}_\phi\right] \tag{26.70}$$

The velocity $\mathbf{v}(\rho,\phi,z,t)$ in spherical coordinates is

$$\mathbf{v}(r,\theta,\phi,t) = \left[\frac{\hbar}{m_e r_0 \sin\theta}\delta(r - r_0)\,\mathbf{i}_\phi\right] \tag{26.71}$$

The total angular momentum of the scattered electron is given by integration over the two dimensional negatively curved surface having the angular momentum density in spherical coordinates given by

$$\mathbf{Li}_z = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{m_e}{\frac{8}{3}\pi r_0^3}\frac{\hbar}{m_e r_0} r^2 \sin^2\theta\,\delta(r - r_0) r^2 \sin\theta\, dr\, d\theta\, d\phi \tag{26.72}$$

$$\mathbf{Li}_z = \hbar \tag{26.73}$$

where $\rho_o = r_o$.

The electron orbitsphere of an atom has a constant velocity as a function of angle. ***Whereas, the electron orbitsphere formed when the radius of the incoming electron is equal to the radius of the scattering atom (i.e. $z_o = \rho_o$) has a velocity function whose magnitude is harmonic in theta (Eq. (26.71)). The velocity function (Eq. (26.63) or Eq. (26.71)) is a hyperboloid. It exists on a two dimension sphere; thus, it is spatially bounded. The mass and charge functions given by Eq. (26.68) and Eq. (26.69), respectively, are finite on a two dimensional sphere; thus, they are bounded. The scattered electron having a negatively curved two dimensional velocity surface is called a hyperbolic electron.*** The magnetic field of the current-density function of the hyperbolic electron provides the force balance of the centrifugal force of the mass density function as was the case for the free electron given in the Electron in Free Space Section. The current density function is also nonradiative as given in that section. Hyperbolic electrons can be focused into a beam by electric and/or magnetic fields to form a hyperbolic electron beam. The velocity distribution along the z-axis of a hyperbolic electron is shown schematically in Figure 26.4A. A cutaway of the velocity distribution of a hyperbolic electron is shown schematically in Figure 26.4B.

Figure 26.4A. The magnitude of the velocity distribution ($|\mathbf{v}_\phi|$) on a two dimension sphere along the z-axis (vertical axis) of a hyperbolic electron.

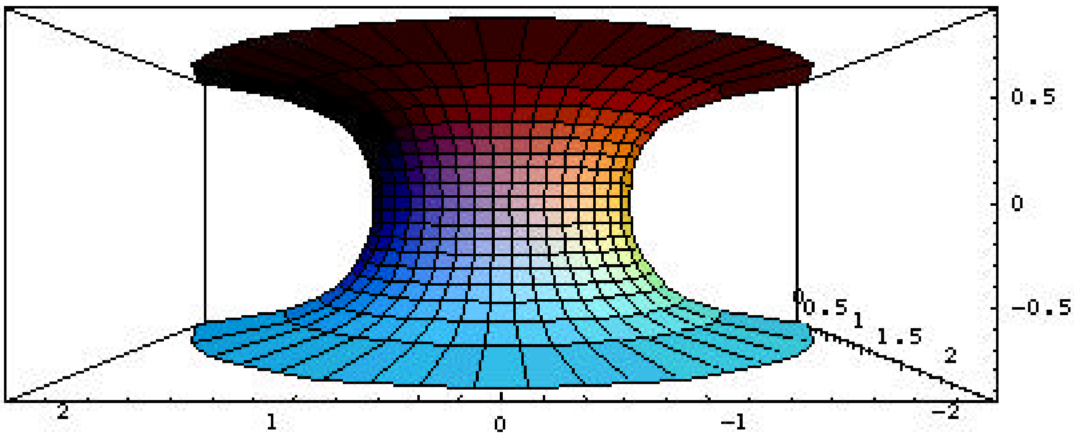
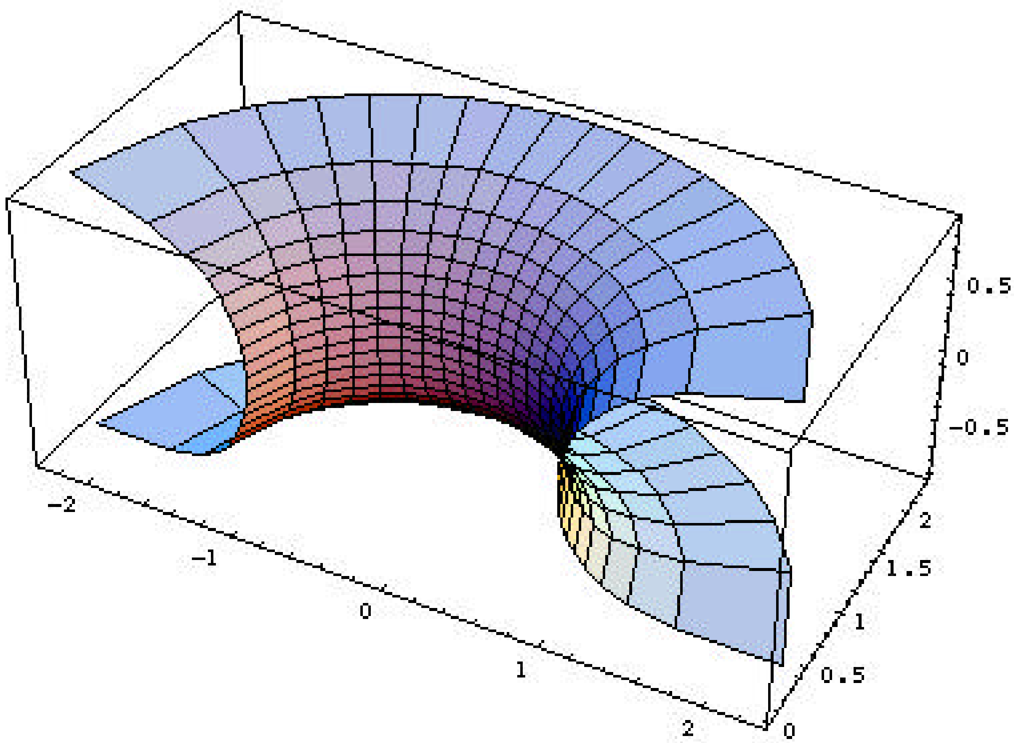


Figure 26.4B. A cutaway of the magnitude of the velocity distribution ($|\mathbf{v}_\phi|$) on a two dimension sphere along the z-axis (vertical axis) of a hyperbolic electron.



The velocity is harmonic or imaginary as a function of theta. Therefore, the gravitational velocity of the Earth relative to that of the hyperbolic electron is imaginary. This case corresponds to an eccentricity greater than one and a hyperbolic orbit of Newton's Law of Gravitation. The metric for the imaginary gravitational velocity is derived based on the center of mass of the scattering event. The Earth, helium, and the hyperbolic electron are spherically symmetrical; thus, the Schwarzschild metric (Eqs. (26-41-26.42)) applies. The velocity distribution defines a surface of negative curvature relative to the positive curvature of the Earth. This case corresponds to a negative radius of Eq. (26.40) or an imaginary gravitational velocity of Eq. (26.36). The lift due to the resulting antigravitational force is given in the Preferred Embodiment of an Antigravitational Device Section. According to Eq. (23.48) and Eq. (23.140), matter, energy, and spacetime are conserved with respect to creation of a particle which is repelled from a gravitating body. The gravitationally ejected particle gains energy as it is repelled. The ejection of a particle having a negatively curved velocity surface such as a hyperbolic electron from a gravitating body such as the Earth must result in an infinitesimal decrease in its radius of the gravitating body (e.g. r of the Schwarzschild metric given by Eq. (26.2) where $m_0 = M$ is the mass of the Earth). The amount that the gravitational potential energy of the gravitating body is lowered is equivalent to the energy gained by the repelled particle. The physics is time reversible. The process may be run backwards to achieve the original state before the repelled particle such as a hyperbolic electron was created.

In a preferred embodiment, the neutral atoms of the neutral atom beam comprise helium, and the velocity of the free electrons of the electron beam is

$$v_z = \frac{\hbar}{m_e \rho_o} = 3.858361\ X\ 10^6\ m/s \tag{26.74}$$

where $\rho_o = 0.567 a_o = 3.000434\ X\ 10^{-11}\ m$.

In another preferred embodiment, each atom of the neutral atomic beam comprises hydrino atom $H(1/p)$, $\rho_o = \frac{a_H}{p}$; p is an integer). The velocity of each electron of the free electron beam is

$$v_z = \frac{\hbar}{m_e \rho_o} = 2.187691\ X\ 10^6\ m/s \tag{26.75}$$

where $\rho_o = \frac{a_H}{n} = \frac{5.29177\ X\ 10^{-11}}{n}\ m$

For a nonrelativistic electron of velocity v_z, the kinetic energy, T, is

$$T = \frac{1}{2} m_e v_z^2 \qquad (26.76)$$

In the case of helium with the substitution of Eq. (26.74) into Eq. (26.76),

$$T = 42.3\ eV \qquad (26.77)$$

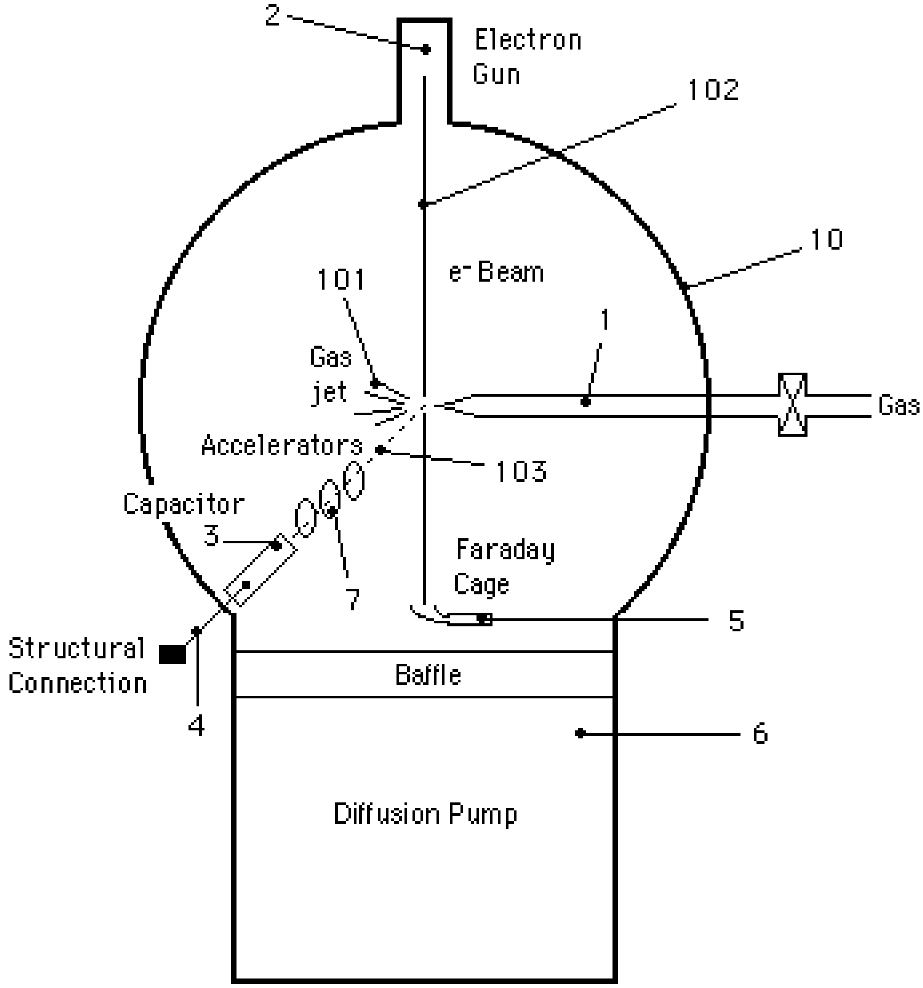
In the case of hydrogen with the substitution of Eq. (26.75) into Eq. (26.76),

$$T = p^2 13.6\ eV \qquad (26.78)$$

PREFERRED EMBODIMENT OF AN ANTIGRAVITY DEVICE

As shown schematically in Figure 26.5, the device 10 of Mills [5] to provide an antigravitational force for levitation or propulsion comprises a source 1 of a gas jet of atoms 101 such as helium atoms such as described by Bonham [6] and an energy tunable electron beam source 2 which supplies an electron beam 102 having electrons of a precise energy such that the radius of each electron is equal to the radius of each atom of the gas jet 101. Such a source is described by Bonham [6]. The gas jet 101 and electron beam 102 intersect such that the velocity function of each electron is elastically scattered and warped into a hyperboloid of negative curvature (hyperbolic electron). The hyperbolic electron beam 103 passes into an electric field provided by a capacitor means 3. In a preferred embodiment, the capacitor means 3 is along to the electron beam 102, and the intersection of the gas jet 101 and the electron beam 102 occurs inside of the capacitor means 3. The hyperbolic electrons experience an antigravitational force due to their velocity surfaces of negative curvature and are accelerated away from the center of the gravitating body such as the Earth. This upward force is transferred to the capacitor means 3 via a repulsive electric force between the hyperbolic electrons and the electric field of the capacitor means 3. The capacitor means 3 is rigidly attached to the body to be levitated or propelled by structural attachment 4. The present antigravity means further includes a means to trap unscattered and hyperbolic electrons and recirculate them through the beam 102. Such a trap means 5 includes a Faraday cage as described by Bonham [6]. The present antigravity means 10 further includes a means 6 to trap and recirculate the atoms of the gas jet 101. Such a gas trap means 6 includes a pump such as a diffusion pump as described by Bonham [6].

Figure 26.5. Antigravity device driven by hyperbolic electrons.



In the case of a hyperbolic electron which is much smaller then the size of a capacitor, the electric force of the hyperbolic electron on the capacitor is equivalent to that of a point charge. This force provides lift to the capacitor due to the gravitational repulsion of the hyperbolic electron from the Earth as it undergoes a trajectory through the capacitor. A close approximation of the trajectory of hyperbolic electrons generated by the antigravity levitation and propulsion means can be found by solving the Newtonian inverse-square gravitational force equations for the case of a repulsive force. The trajectory follows from the Newtonian gravitational force and the solution of motion in an inverse-square repulsive field given by Fowles [7]. The trajectory can be calculated rigorously by solving the orbital equation from the Schwarzschild metric (Eqs. (26.15-26.16)) for a two-dimensional spatial velocity density function of negative curvature which is produced by the apparatus and repelled by the Earth. The rigorous solution is equivalent to that given for the case of a positive gravitational velocity given in the Orbital Mechanics Section except that the gravitational velocity is imaginary, or the gravitational radius is negative.

In the case of a velocity function having negative curvature, Eq. (23.78) becomes

$$\left(1+\frac{2GM}{rc^2}\right)\frac{dt}{d\tau}=\frac{E}{mc^2} \tag{26.79}$$

where M is the mass of the Earth and m is the mass of the hyperbolic electron. Eq. (23.79) is based on the equations of motion of the geodesic, which in the case of an imaginary gravitation velocity or a negative gravitational radius becomes

$$\left(\frac{dr}{d\theta}\right)^2=\frac{r^4}{L_\theta^2}\left[\left(\frac{E}{c}\right)^2-\left(1+\frac{2GM}{c^2r}\right)\left(\frac{L_\theta^2}{r^2}+m^2c^2\right)\right] \tag{26.80}$$

The repulsive central force equations can be transformed into an orbital equation by the substitution, $u=\frac{1}{r}$. The relativistically corrected differential equation of the orbit of a particle moving under a repulsive central force is

$$\left(\frac{du}{d\theta}\right)^2+u^2=\frac{\left(\frac{E}{c}\right)^2-m^2c^2}{L_\theta^2}-\frac{m^2c^2}{L_\theta^2}\left(\frac{2GM}{c^2}\right)u-\left(\frac{2GM}{c^2}\right)u^3 \tag{26.81}$$

By differentiating with respect to θ, noting that $u=u(\theta)$ gives

$$\frac{d^2u}{d\theta^2}+u=-\frac{GM}{a^2}-\frac{3}{2}\left(\frac{2GM}{c^2}\right)u^2 \tag{26.82}$$

where

$$a=\frac{L_\theta}{m} \tag{26.83}$$

In the case of a weak field,

$$\left(\frac{2GM}{c^2}\right)u<<1 \tag{26.84}$$

and the second term on the right-hand of (26.37) can then be neglected in the zero-order. The equation of the orbit is

$$u_0=\frac{1}{r}=A\cos(\theta+\theta_0)-\frac{GM}{a^2} \tag{26.85}$$

$$r=\frac{1}{A\cos(\theta+\theta_0)-\frac{GM}{a^2}} \tag{26.86}$$

where A and θ_0 denote the constants of integration. Consider E_t, the sum of the kinetic and gravitational potential energy:

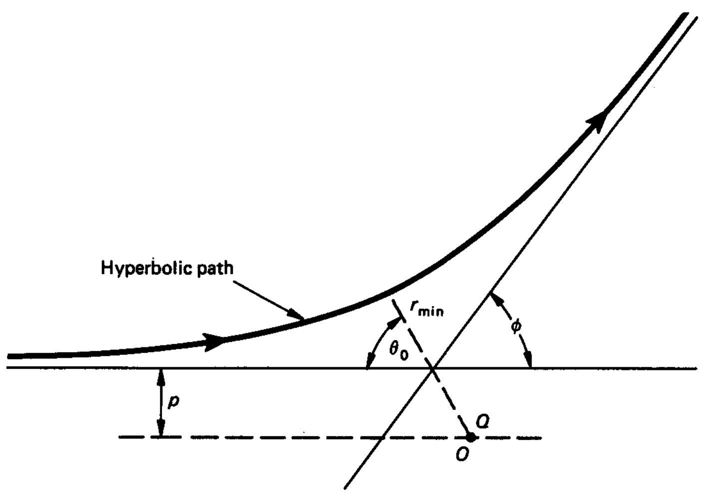
$$E_t=\frac{1}{2}mv^2+\frac{GMm}{r} \tag{26.87}$$

where m is the mass of the hyperbolic electron. The orbit equation may also be expressed in terms of E_t as given by Fowles [8]

$$r = \frac{\frac{\alpha^2}{GM}}{-1 + \left(1 + \frac{2E m \alpha^2}{(GMm)^2}\right)^{\frac{1}{2}} \cos(\theta - \theta_0)} \tag{26.88}$$

In a repulsive field, the energy is always greater than zero. Thus, the eccentricity e, the coefficient of $\cos(\theta - \theta_0)$, must be greater than unity ($e > 1$) which requires that the orbit must be hyperbolic. Consider the trajectory of a hyperbolic electron shown in Figure 26.6.

Figure 26.6. Hyperbolic path of a hyperbolic electron of mass m in an inverse-square repulsive field of a gravitating body comprised of positively curved matter of total mass M.



It approaches along one asymptote and recedes along the other. The direction of the polar axis is selected such that the initial position of the hyperbolic electron is $\theta = 0$, $r = \infty$. According to either of the equations of the orbit (Eq. (26.86) or Eq. (26.88)) r assumes its minimum value when $\cos(\theta - \theta_0) = 1$, that is, when $\theta = \theta_0$. Since $r = \infty$ when $\theta = 0$, then r is also infinite when $\theta = 2\theta_0$. Therefore, the angle between the two asymptotes of the hyperbolic path is $2\theta_0$, and the angle ϕ through which the incident hyperbolic electron is deflected is given by

$$\phi = \pi - 2\theta_0 \tag{26.89}$$

Furthermore, the denominator of Eq. (26.88) vanishes when $\theta = 0$ and $\theta = 2\theta_0$. Thus,

$$-1+\left(1+\frac{2Ema^2}{(GMm)^2}\right)^{-\frac{1}{2}}\cos(\theta_0)=0 \tag{26.90}$$

Using Eq. (26.89) and Eq. (26.90), the scattering angle ϕ is given in terms of θ as

$$\tan\theta_0=\frac{(2Em)^{\frac{1}{2}}a}{GMm}=\cot\frac{\phi}{2} \tag{26.91}$$

For convenience, the constant $a=\frac{L_\theta}{m}$ may be expressed in terms of another parameter p called the impact parameter. The impact parameter is the perpendicular distance from the origin (deflection or scattering center) to the initial line of motion of the hyperbolic electron as shown in Figure 26.6. The relationship between a the angular momentum per unit mass and v_0 the initial velocity of the hyperbolic electron is

$$a=|\mathbf{r}\times\mathbf{v}|=pv_0 \tag{26.92}$$

A massive gravitational body such as the Earth will not be moved by the encounter with a hyperbolic electron. Thus, the energy E_t of the deflected hyperbolic electron is constant and is equal to T the initial kinetic energy because the initial potential energy is zero ($r=\infty$).

$$T=\frac{1}{2}mv_0^2 \tag{26.93}$$

Using the impact parameter, the deflection or scattering equation is given by

$$\cot\frac{\phi}{2}=\frac{pv_0^2}{GM}=\frac{2pE}{GMm} \tag{26.94}$$

$$\phi=2\arctan\left(\frac{pv_0^2}{GM}\right)^{-1}=2\arctan\left(\frac{2pE}{GMm}\right)^{-1} \tag{26.95}$$

The gravitational velocity of the Earth v_{g_E} is approximately

$$v_{g_E}\approx\sqrt{\frac{2GM}{p}} \tag{26.96}$$

Thus, Eq. (26.95) is given by

$$\phi=2\arctan\left(\frac{1}{2}\left(\frac{v_{g_E}}{v_0}\right)^2\right) \tag{26.97}$$

Consider the postulate that the hyperbolic electron must follow the trajectory for an inverse squared force in the far field. In the limit, the far field trajectory is the asymptote. As a method to obtain a first approximation of the asymptote, consider the case that the hyperbolic electron is generated at the surface of the Earth with an initial trajectory as shown in Figure 26.6. The initial radial position is $r_{\min}$ which is the radius of the Earth. Also, the impact parameter p is essentially equal to the radius of the Earth. Substitution of Eq. (26.87) and Eq. (26.92) into Eq. (26.91) gives

$$\frac{\left(v_0^2 + \frac{2GM}{p}\right)^{\frac{1}{2}} p v_0}{GM} = \cot\frac{\phi}{2} \qquad (26.98)$$

Substitution of Eq. (26.96) into Eq. (26.98) gives

$$2\left(v_0^2 + v_{g_E}^2\right)^{\frac{1}{2}} \frac{v_0}{v_{g_E}^2} = \cot\frac{\phi}{2} \qquad (26.99)$$

$$\phi = 2\arctan\left(\frac{1}{2}\frac{v_{g_E}^2}{\left(v_0^2 + v_{g_E}^2\right)^{\frac{1}{2}} v_0}\right) \qquad (26.100)$$

The gravitational velocity of the Earth v_{g_E} is

$$v_{g_E} = \sqrt{\frac{2GM}{R}} = 1.1\ X\ 10^8\ m/\sec \qquad (26.101)$$

where R is the radius of the Earth. Consider the case of the generation of hyperbolic electrons via elastic scattering from helium atoms. Substitution of the hyperbolic electron velocity of $2.187691\ X\ 10^6\ m/s$ given by Eq. (26.75) and the gravitational velocity of the Earth given by Eq. (26.101) into Eq. (26.100) gives

$$\phi = 2\arctan\left(\frac{1}{2}\frac{\left(1.1\ X\ 10^8\ m/\sec\right)^2}{\left(\left(1.1\ X\ 10^8\ m/\sec\right)^2 + \left(2.2\ X\ 10^6\ m/\sec\right)^2\right)^{\frac{1}{2}}\left(2.2\ X\ 10^6\ m/\sec\right)}\right) \qquad (26.102)$$

The angle of the asymptote is

$$\phi = 175° \approx \pi \qquad (26.103)$$

Thus, the asymptote of the trajectory of a hyperbolic electron is essentially radial from the Earth. Since the trajectory in a conservative inverse field is reversible going from $+\infty$ to $-\infty$ or vice versa, the entire trajectory of a hyperbolic electron with $v_0 = 2.187691\ X\ 10^6\ m/s$ at $r_{\min}$ equal to the radius of the Earth is essentially radial with respect to the Earth. From this result, it can be concluded that the far field trajectory of a hyperbolic electron formed from a free electron with an initial kinetic energy of $42.3\ eV$ and an initial electron velocity of $2.187691\ X\ 10^6\ m/s$ in an arbitrary initial direction relative to the Earth is essentially radial from the Earth since 1.) v_0 is much less than v_{g_E}, 2.) the impact parameter is essentially $r_{\min}$ which is the radius of the Earth since the radius of the

Earth is so large, and 3.) the free electron has zero gravitational mass. The trajectory forms the gravitational boundary condition to be matched with the additional scattering boundary condition.

The scattering distribution of hyperbolic electrons given by Eq. (26.51) is centered at a scattering angle of Θ_o given by Eq. (26.54). With the condition $z_o = \rho_o$, the elastic electron scattering angle in the far field Θ_o is determined by the boundary conditions of the curvature of spacetime due to the presence of a gravitating body and the constant maximum velocity of the speed of light. The far field condition must be satisfied with respect to electron scattering and the gravitational orbital equation. The former condition is met by Eq. (26.51) and Eq. (26.54). The latter is met by Eq. (26.103) where the far field angle of the hyperbolic gravitational trajectory ϕ is equivalent to Θ_o.

The elastic scattering condition is possible due to the large mass of the helium atom and the Earth relative to the electron wherein the recoil energy transferred during a collision is inversely proportional to the mass as given by Eq. (2.70). Satisfaction of the far field conditions of the elastic electron scattering to produce hyperbolic electrons and the hyperbolic gravitational trajectory requires that the hyperbolic electrons elastically scatter in a direction radially from the Earth with a kinetic energy in the radial direction that is essentially equal to the initial kinetic energy corresponding to the condition $z_o = \rho_o$.

According to Eq. (23.48) and Eq. (23.140), matter, energy, and spacetime are conserved with respect to creation of the hyperbolic electron which is repelled from a gravitating body, the Earth. The gravitationally ejected hyperbolic electron gains energy as it is repelled ($>10^4\ eV$). The ejection of a hyperbolic electron having a negatively curved velocity surface from the Earth must result in an infinitesimal decrease in its radius of the Earth (e.g. r of the Schwarzschild metric given by Eq. (26.2) where $m_0 = M$ is the mass of the Earth). The amount that the gravitational potential energy of the Earth is lowered is equivalent to the energy gained by the repelled hyperbolic electron.

Momentum is also conserved for the electron, Earth, and helium atom wherein the gravitating body that repels the hyperbolic electron, the Earth, receives an equal and opposite change of momentum with respect to that of the electron.

Causing a satellite to follow a hyperbolic trajectory about a gravitating body is a common technique to achieve a gravity assist to further propel the satellite. In this case, the energy and momentum gained by the satellite is also equal and opposite that lost by the gravitating body.

The kinetic energy of the hyperbolic electron corresponding to a velocity of $2.187691 \; X \; 10^6 \; m/s$ is $T = 42.3 \; eV$. Thus, $42.3 \; eV$ may be imparted to the antigravity device per hyperbolic electron. With a beam current of 10^5 amperes achieved in one embodiment by multiple beams such as 100 beams each providing 10^3 amperes, the power transferred to the device P_{AG} is

$$P_{AG} = \frac{10^5 \text{ coulomb}}{\text{sec}} \text{ X } \frac{1 \text{ electron}}{1.6 \text{ X } 10^{-19} \text{ coulombs}} \text{ X } \frac{42.3 \text{ eV}}{\text{electron}} \text{ X } \frac{1.6 \text{ X } 10^{-19} \text{J}}{\text{eV}} = 4.2 \text{ MW} \quad (26.104)$$

The power dissipated against gravity P_G is given by

$$P_G = m_c g v_c \quad (26.105)$$

where m_c is the mass of the craft, g is the acceleration of gravity, v_c is the velocity of the craft. In the case of a $10^4 \; kg$ craft, the 4.2 MW of power provided by Eq. (26.104) sustains a steady lifting velocity of 43 m/sec. Thus, significant lift is possible using hyperbolic electrons.

In the case of a $10^4 \; kg$ craft, F_g, the gravitational force is

$$F_g = m_c g = \left(10^4 \; kg\right)\left(9.8 \; \frac{m}{\sec^2}\right) = 9.8 \; X \; 10^4 \; N \quad (26.106)$$

where m_c is the mass of the craft and g is the standard gravitational acceleration. The lifting force may be determined from the gradient of the energy which is approximately the energy dissipated divided by the vertical (relative to the Earth) distance over which it is dissipated. The antigravitational force provided by the hyperbolic electrons may be controlled by adjusting the electric field of the capacitor. For example, the electric field of the capacitor may be increased such that the levitating force overcomes the gravitational force. In an embodiment of the capacitor, the electric field, E_{cap}, is constant and is given by the capacitor voltage, V_{cap}, divided by the distance between the capacitor plates, d, of a parallel plate capacitor.

$$E_{cap} = \frac{V_{cap}}{d} \quad (26.107)$$

In the case that V_{cap} is $10^6 \; V$ and d is $1 \; m$, the electric field is

$$E_{cap} = \frac{10^6 \; V}{m} \quad (26.108)$$

The force of the electric field of the capacitor on a hyperbolic electron, F_{ele}, is the electric field, E_{cap}, times the fundamental charge

$$F_{ele} = eE_{cap} = \left(1.6 \; X \; 10^{-19} \; C\right)\left(10^6 \; \frac{V}{m}\right) = 1.6 \; X \; 10^{-13} \; N \quad (26.109)$$

The distance traveled away from the Earth, Δr_z, by a hyperbolic electron having an energy of $E = 42.3 \; eV = 6.77 \; X \; 10^{-18} \; J$ is given by the energy divided by the electric field F_{ele}

$$\Delta r_z = \frac{E}{F_{ele}} = \frac{6.77\ X\ 10^{-18}\ J}{1.6\ \ X\ 10^{-13}\ N} = 4.23\ X\ 10^{-5}\ m = 0.0423\ mm \tag{26.110}$$

The number of electrons N_e is given by

$$N_e = \frac{I}{e v_e r_i} \tag{26.111}$$

where I is the current, e is the fundamental electron charge, v_e is the hyperbolic electron velocity, r_i is the length of the current. Substitution of $I = 10^5\ A$, $v_e = v_0 = 2.187691\ X\ 10^6\ m/s$, and $r_i = 0.2\ m$, the number of electrons is

$$N_e = 1.5\ X\ 10^{18} \text{ electrons} \tag{26.112}$$

The antigravitational force, F_{AG}, is given by multiplying the number of electrons (Eq. (26.112)) by the force per electron (Eq. (26.109)).

$$F_{AG} = N_e F_e = \left(1.5\ X\ 10^{18} \text{ electrons}\right)\left(1.6\ X\ 10^{-13}\ N\right) = 2.4\ X\ 10^5\ N \tag{26.113}$$

Thus, the present example of an antigravity device may provide a levitating force that is capable of overcoming the gravitational force on the craft to achieve a maximum vertical velocity of 43 m/sec as given by Eq. (26.105). In an embodiment of the antigravity device, the hyperbolic electron current and the electric field of the capacitor are adjusted to control the vertical acceleration and velocity.

Levitation by an antigravitational force is orders of magnitude more energy efficient than conventional rocketry. In the former case, the energy dissipation is converted directly to gravitational potential energy as the craft is lifted out of the gravitation field. Whereas, in the case of rocketry, matter is expelled at a higher velocity than the craft to provide thrust or lift. The basis of rocketry's tremendous inefficiency of energy dissipation to gravitational potential energy conversion may be determined from the thrust equation. In a case wherein external forces including gravity are taken as zero for simplicity, the thrust equation is [9]

$$v = v_0 + V \ln \frac{m_0}{m} \tag{26.114}$$

where v is the velocity of the rocket at any time, v_0 is the initial velocity of the rocket, m_0 is the initial mass of the rocket plus unburned fuel, m is the mass at any time, and V is the speed of the ejected fuel relative to the rocket. Owing to the nature of the logarithmic function, it is necessary to have a large fuel to payload ratio in order to attain the large speeds needed for satellite launching, for example.

The antigravitational force of hyperbolic electrons can be increased by using atoms of the neutral atom beam of relativistic kinetic energy. The electrons of the electron beam and the relativistic atoms of the neutral atomic beam intersect at an angle such that the relativistically contracted radius of each atom, z_o, is equal to ρ_o, the radius of each free electron of the electron beam. Elastic scattering produces hyperbolic electrons at relativistic energies. The relativistic radius of helium is calculated by substitution of the relativistic mass (Eq. (24.14)) of helium

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{26.115}$$

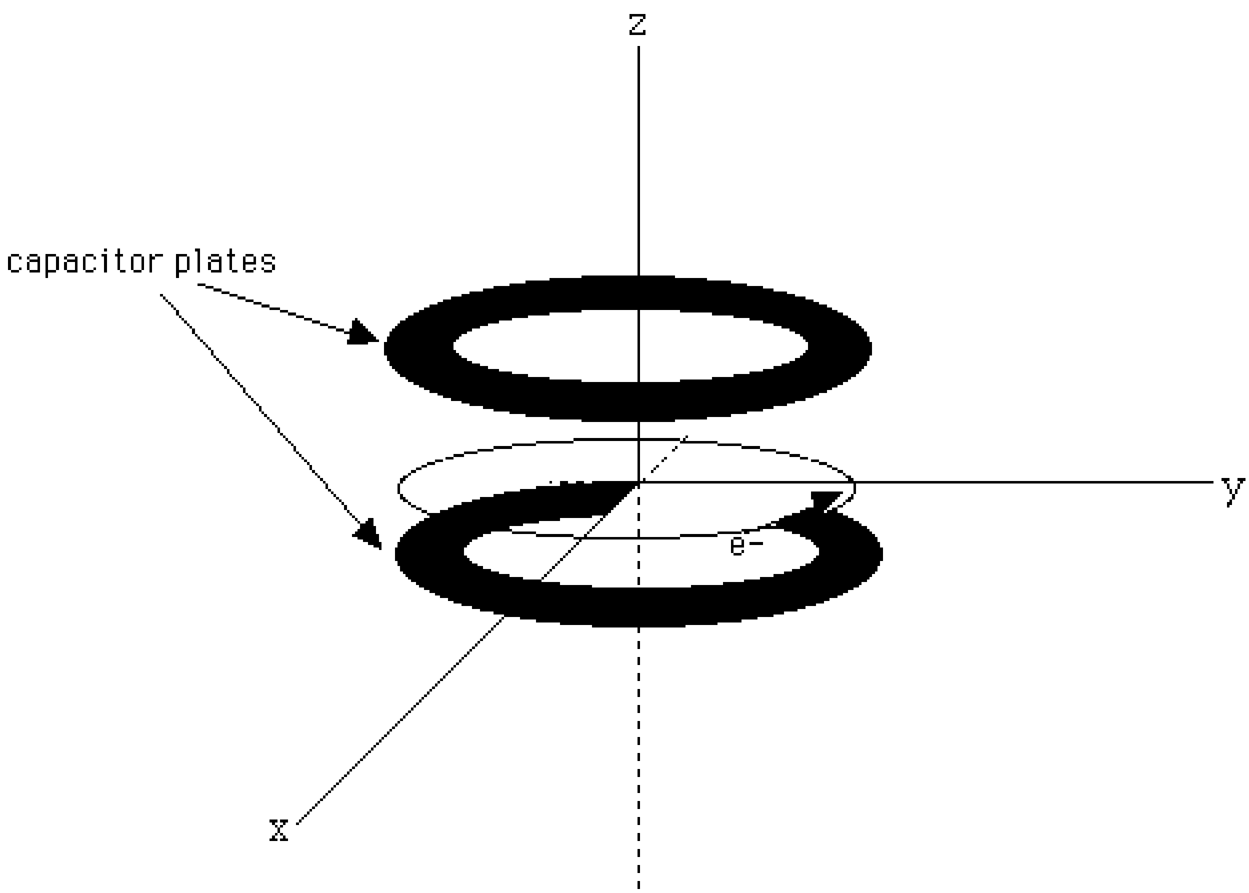
into Eq. (7.19) with a_o given by Eq. (1.168) where Eq. (26.115) is transformed from Cartesian coordinates to spherical coordinates. In a preferred embodiment, the relativistic atomic beam which intersects the electron beam directed along the negative x-axis is oriented at an angle of $\frac{\pi}{4}$ to both the xz and yz-planes with the relativistic radius of each neutral atom equal to the radius of each free electron.

In another embodiment, high energy hyperbolic electrons are created by scattering according to Eq. (26.75) and Eq. (26.78) from hydrino atoms of small radii. Since hydrino atoms form hydrino hydride ions for $p \leq 24$, hydrino atoms of $p > 24$ are preferably used.

In another embodiment shown in Figure 26.5, hyperbolic electrons are accelerated to relativistic energies by an acceleration means 7 before entering or within the capacitor means 3 to provide relativistic hyperbolic electrons with increased energy to be converted to gravitational potential energy as the body to be levitated is levitated.

In the case of relativistic hyperbolic electrons, the distance traveled in order to transfer a substantial amount of the kinetic energy of the hyperbolic electron to an axis parallel to that of the radius of the Earth is much greater than the case of low hyperbolic electron velocities. With a relativistic hyperbolic electron initially propagating in the direction perpendicular to the radius of the Earth, a path length of many meters may be required for the hyperbolic electron to act on the capacitor. In one embodiment of the antigravity device, a capacitor may further comprise a synchrotron for forcing the hyperbolic electron in a orbit with a component of the velocity in the xy-plane such as that shown in Figure 26.7 which is perpendicular to the radius of the Earth. The hyperbolic electron held in a synchrotron orbit in the xy-plane is repelled by the Earth and transfers a force to the capacitor in the z direction as shown in Figure 26.7.

Figure 26.7. Helical motion of a hyperbolic electron in a synchrotron orbit in the xy-plane with an antigravitational acceleration along the +z axis which is transferred to the capacitor.



MECHANICS

In addition to levitation, acceleration in a direction tangential to the gravitating body's surface can be effected via conservation of angular momentum. Thus, a radially accelerated structure such as an aerospace vehicle to be tangentially accelerated possesses a cylindrically or spherically symmetrically movable mass having a moment of inertia, such as a flywheel device. The flywheel is rotated by a driving device which provides angular momentum to the flywheel. Such a device is the electron beams which are the source of hyperbolic electrons. The electrons move rectilinearly until being elastically scattered from an atomic beam to form hyperbolic electrons which are deflected in a radial direction from the center of the gravitating body. A component to the initial momentum of the electron beam is transferred to the gravitating body as the hyperbolic electrons are deflected upward by the gravitating body. The opposite momentum is transferred to the source of the electron beam. This momentum may be used to translate the craft in a direction tangential to the gravitating body's surface or to cause it to spin. Thus, the electron beam serves the additional function of a source of transverse or angular acceleration. Thus, it may be considered an ion rocket.

The vehicle is levitated using antigravity means to overcome the gravitational force of the gravitating body where the levitation is such that the angular momentum vector of the flywheel is parallel to the radial or central vector of the gravitational force of the gravitating body. The angular momentum vector of the flywheel is forced to make a finite angle with the radial vector of gravitational force by tuning the symmetry of the levitating (antigravitational) forces provided by an antigravity apparatus comprising multiple elements at different spatial locations of the vehicle. A torque is produced on the flywheel as the angular momentum vector is reoriented with respect to the radial vector due to the interaction of the central force of gravity of the gravitating body, the force of antigravity of the antigravity means, and the angular momentum of the flywheel device. The resulting acceleration which conserves angular momentum is perpendicular to the plane formed by the radial vector and the angular momentum vector. Thus, the resulting acceleration is tangential to the surface of the gravitating body.

Large translational velocities are achievable by executing a trajectory which is vertical followed by a precession with a large radius that gives a translation to the craft. The latter motion is effected by tilting the spinning craft to cause it to precess with a radius that increases due to the force provided by the craft acting as an airfoil. The tilt is provided by the activation and deactivation of multiple antigravitational devices spaced so that the desired torque perpendicular to the spin axis is maintained. The craft also undergoes a controlled fall and gains a velocity that provides the centrifugal force to the precession as the craft acts as an airfoil. During the translational acceleration, energy stored in the flywheel is converted to kinetic energy of the vehicle. As the radius of the precession goes to infinity the rotational energy is entirely converted into transitional kinetic energy. The equation for rotational kinetic energy E_R and translational kinetic energy E_T are given as follows:

$$E_R = \frac{1}{2} I\omega^2 \tag{26.116}$$

where I is the moment of inertia and ω is the angular rotational frequency;

$$E_T = \frac{1}{2} m v^2 \tag{26.117}$$

where m is the total mass and v is the translational velocity of the craft. The equation for the moment of inertia I of the flywheel is given as:

$$I = \sum m_i r^2 \tag{26.118}$$

where m_i is the infinitesimal mass at a distance r from the center of mass. Eqs. (26.116) and (26.118) demonstrate that the rotational kinetic energy stored for a given mass is maximized by maximizing the distance of the mass from the center of mass. Thus, ideal design parameters are cylindrical symmetry with the rotating mass, flywheel, at the perimeter of the vehicle.

The equation that describes the motion of the vehicle with a moment of inertia I, a spin moment of inertial I_s, a total mass m, and a spin frequency of its flywheel of S is given as follows [10]:

$$mgl \sin\theta = I\ddot{\theta} + I_s S \dot{\phi} \sin\theta - I\dot{\phi}^2 \cos\theta \sin\theta \tag{26.119}$$

$$0 = I \frac{d}{dt}\left(\dot{\phi} \sin\theta\right) - I_s S \dot{\theta} + I \dot{\theta}\dot{\phi} \cos\theta \tag{26.120}$$

$$0 = I_s \dot{S} \tag{26.121}$$

where θ is the tilt angle between the radial vector and the angular momentum vector, $\ddot{\theta}$ is the acceleration of the tilt angle θ, g is the acceleration due to gravity, l is the height to which the vehicle levitates, and $\dot{\phi}$ is the angular precession frequency resulting from the torque which is a consequence of tilting the craft. Eq. (26.121) shows that S, the spin of the craft about the symmetry axis, remains constant. Also, the component of the angular momentum along that axis is constant.

$$L_z = I_s S = \text{constant} \tag{26.122}$$

Eq. (26.120) is then equivalent to

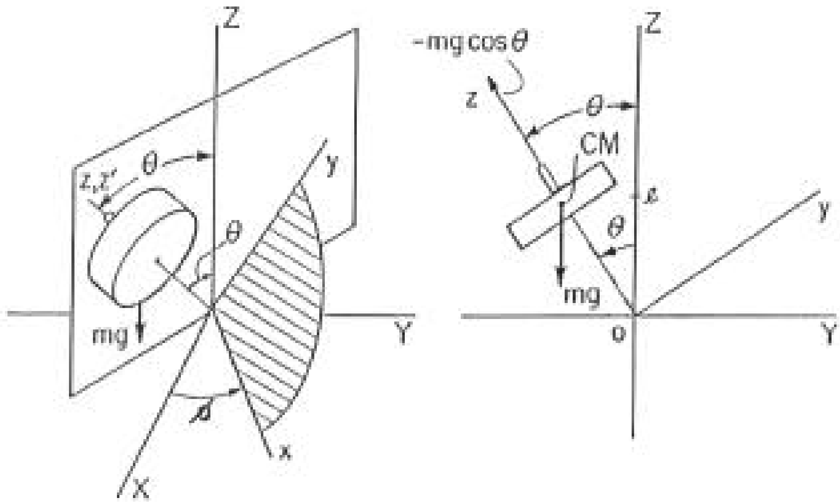
$$0 = \frac{d}{dt}\left(I\dot{\phi} \sin^2\theta + I_s S \cos\theta\right) \tag{26.123}$$

so that

$$I\dot{\phi} \sin^2\theta + I_s S \cos\theta = B = \text{constant} \tag{26.124}$$

The craft is an airfoil which provides the centrifugal force to move the center of mass of the craft away from the Z axis of the stationary frame. The schematic appears in Figure 26.8.

Figure 26.8. Schematic of the forces on a spinning craft which is caused to tilt.



If there is no drag acting on the spinning craft to dissipate its energy E, then the total energy E equal to the kinetic T and potential V remains constant:

$$\frac{1}{2}\left(I\omega_x^2 + I\omega_y^2 + I_s S^2\right) + mgl\cos\theta = E \tag{26.125}$$

or equivalently in terms of Eulerian angles,

$$\frac{1}{2}\left(I\dot{\theta}^2 + I\dot{\phi}^2\sin^2\theta + I_s S^2\right) + mgl\cos\theta = E \tag{26.126}$$

From Eq. (26.124), $\dot{\phi}$ may be solved and substituted into Eq. (26.126). The result is

$$\frac{1}{2}I\dot{\theta}^2 + \frac{\left(B - I_s S\cos\theta\right)^2}{2I\sin^2\theta} + \frac{1}{2}I_s S^2 + mgl\cos\theta = E \tag{26.127}$$

which is entirely in terms of θ. Eq. (26.126) permits θ to be obtained as a function of time t by integration. The following substitution may be made:

$$u = \cos\theta \tag{26.128}$$

Then

$$\dot{u} = -(\sin\theta)\dot{\theta} = -\left(1-u^2\right)^{1/2}\dot{\theta} \tag{26.129}$$

Eq. (26.127) is then

$$\dot{u}^2 = \left(1-u^2\right)\left(2E - I_s S^2 - 2mglu\right)I^{-1} - \left(B - I_s Su\right)^2 I^{-2} \tag{26.130}$$

or

$$\dot{u}^2 = f(u) \tag{26.131}$$

from which u (hence θ) may be solved as a function of t by integration:

$$t = \int \frac{du}{\sqrt{f(u)}} \tag{26.132}$$

In Eq. (26.132), $f(u)$ is a cubic polynomial, thus, the integration may be carried out in terms of elliptic functions. Then the precession velocity $\dot{\phi}$ may be solved may be solved by substitution of θ into Eq. (26.124) wherein the constant B is the initial angular momentum of the craft along the spin axis, I_sS given by Eq. (26.122). The radius of the precession is given by

$$R = l\sin\theta \tag{26.133}$$

And the linear velocity v of the precession is given by

$$v = R\dot{\phi} \tag{26.134}$$

The maximum rotational speed for steel is approximately $1100\ m/\sec$ [11]. For a craft with a radius of $10\ m$, the corresponding angular velocity is $\frac{110\ cycles}{\sec}$. In the case that most of the mass of a $10^4\ kg$ was at this radius, the initial rotation energy (Eq. (26.116)) is $6\ X\ 10^9\ J$. As the craft tilts and changes altitude (increases or decreases), the airfoil pushes the craft away from the axis that is radial with respect to the Earth. For example, as the craft tilts and falls, the airfoil pushes the craft into a trajectory which is analogous to that of a gyroscope as shown in Figure 26.8. From Figure 26.8, the centrifugal force provided by the airfoil ($mg\cos\theta$) is always less than the force of gravity on the craft. From Eq. (26.124), the rotational energy is transferred from the initial spin to the precession as the angle θ increases. From Eq. (26.125), the precessional energy may become essentially equal to the initial rotational energy plus the initial gravitational potential energy. Thus, the linear velocity of the craft may reach approximately $1100\ m/\sec$ ($2500\ mph$). During the transfer, the craft falls approximately one half the distance of the radius of the precession of the center of mass about the Z axis. Thus, the initial vertical height l must be greater.

In the cases of solar system and interstellar travel, velocities approaching the speed of light may be obtained by using gravity assists from massive gravitating bodies wherein the antigravitational capability of the craft establishes the desired trajectory to maximize the assist.

EXPERIMENTAL

The electron-impact energy-loss spectrum of helium taken in the forward direction with $100\ eV$ incident electrons with a resolution of $0.15\ eV$ by Simpson, Mielczarek, and Cooper [12] showed large energy-loss peaks at $57.7\ eV$, $60.0\ eV$, and $63.6\ eV$. Resonances in the photoionization continuum of helium at $60\ eV$ and in the $63.6\ eV$ region have been observed spectroscopically by Madden and Codling [13] using synchrotron radiation. Absent was a resonance at $57.7\ eV$. Both Simpson and Madden assign the peaks of their data to two-electron excitation states in helium. Each of these states decays with the emission of an ionization electron of energy equal to the excitation energy minus the ionization energy of helium, $24.59\ eV$. The data of Goruganthu and Bonham [14] shows ejected-energy peaks at $35.5\ eV$ and at $39.1\ eV$ corresponding to the energy loss peaks of Simpson of $60.0\ eV$ and $63.6\ eV$, respectively. The absence of an ejected-energy peak corresponding to the energy-loss peak at $57.7\ eV$ precludes the assignment of this peak to a two-electron resonance. The energy of each inelastically scattered electron of incident energy of $100\ eV$ corresponding to the energy-loss of $57.7\ eV$ is $42.3\ eV$. This is the resonance energy of hyperbolic electron production by electron scattering from helium given by Eq. (26.77). Thus, the $57.7\ eV$ energy-loss peak of Simpson arises from inelastic scattering of electrons of $42.3\ eV$ from helium with resonant hyperbolic electron production. The production of electrons with velocity functions having negative curvature is experimentally supported.

The electron-impact energy-loss spectrum of helium taken in the forward direction with $400\ eV$ incident electrons by Priestley and Whiddington [15] showed large energy-loss peaks at $42.4\ eV$, and $60.8\ eV$. A resonance in the photoionization continuum of helium at $60\ eV$ has been observed spectroscopically by Madden and Codling [13] using synchrotron radiation. Absent was a resonance at $42.4\ eV$. Both Priestley and Madden assign the peaks of their data to two-electron excitation states in helium. Each of these states decay with the emission of an ionization electron of energy equal to the excitation energy minus the ionization energy of helium, $24.59\ eV$. The data of Goruganthu and Bonham [14] shows an ejected-energy peak at $35.5\ eV$ corresponding to the energy loss peak of Priestley of $60.8\ eV$. The absence of an ejected-energy peak at $17.8\ eV$ corresponding to the energy-loss peak at $42.4\ eV$ precludes the assignment of this peak to a two-electron resonance. This is the resonance energy of hyperbolic electron production by electron scattering from helium given by Eq. (26.77). Thus, the $42.4\ eV$ energy-loss peak of Priestley arises from inelastic scattering of electrons of $42.3\ eV$ from helium with resonant hyperbolic electron production. The production of electrons with velocity functions having negative curvature is experimentally further supported.

References

1. Fock, V., The Theory of Space, Time, and Gravitation, The MacMillan Company, (1964).
2. Fang , L. Z., and Ruffini, R., Basic Concepts in Relativistic Astrophysics,

World Scientific, (1983).
3. Fowles, G. R., Analytical Mechanics, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 154-155.
4. Witteborn, F. C., and Fairbank, W. M., Physical Review Letters, Vol. 19, No. 18, (1967), pp. 1049-1052.
5. R. L. Mills, Apparatus and Method for Providing an Antigravitational Force, PCT Patent Appln. No.: PCT/US95/006140 (1994).
6. Bonham, R. F., Fink, M., High Energy Electron Scattering, ACS Monograph, Van Nostrand Reinhold Company, New York, (1974).
7. Fowles, G. R., Analytical Mechanics, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 140-164.
8. Fowles, G. R., Analytical Mechanics, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 154-160.
9. Fowles, G. R., Analytical Mechanics, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 182-184.
10. Fowles, G. R., Analytical Mechanics, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 243-247.
11. J. W. Beams, "Ultrahigh-Speed Rotation", pp. 135-147.
12. Simpson, J. A., Mielczarek, S. R., Cooper, J., Journal of the Optical Society of America, Vol. 54, (1964), pp. 269-270.
13. Madden, R. B., Codling, K., Astrophysical Journal, Vol. 141, (1965), pp. 364-375.
14. Goruganthu, R. R., Bonham, R. A., Physical Review A, Vol. 34, No. 1, (1986), pp. 103-125.
15. Priestley, H., Whiddington, R., Proc. Leeds Phil. Soc., Vol. 3, (1935), p. 81.