

Einstein's revenge

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To the reader

Origin

The development of the ideas in this book span three decades. In high school I became convinced that no infinite or continuous structures exist. This was reinforced as an undergraduate studying Zermelo Fraenkel set theory (where all of mathematics was built discretely or by induction from the empty set) and computer science and recursive function theory (where all mechanistic physical processes could be built from boolean logic and a *potentially* infinite storage device).

The quantization of energy in physics was further evidence to me of the discrete discontinuous nature of reality. In my first course in quantum mechanics I came to suspect that a discretized finite difference approximation to the wave equation was a good candidate for explaining quantum mechanical effects while preserving Einstein causality.

This same train of thought led me to ask what the discrete structure that underlies physics is. I concluded that the simplest answer was that existence *is the same thing as* consciousness. This idea was elaborated in a paper in an advanced philosophy course.

I tried to develop my ideas about physics for a Ph.D. thesis. This was deemed too risky a topic and von Neumann's false proof[34] that no more complete theory was possible was still widely accepted. Bell's refutation of this proof was published around this time[4] but I was not aware of it. I eventually completed a more conventional thesis in computer science[10].

With one avenue blocked, I pursued another. I investigated the mathematical implications of rejecting completed infinite totalities.

I came to understand that most of the mathematics that is dependent on such structures could be reinterpreted as the mathematics of creativity in a potentially infinite universe. I discussed these ideas at the time with some leading logicians but since there was no new mathematics involved nothing came of these discussions.

Recently I have been able to extend my ideas in physics. The sci.physics newsgroup on Internet has been particularly helpful to me in this. It has allowed me to get a wide variety of feedback on my unconventional ideas and to learn the language of modern quantum mechanics.

The difficulties in getting support to work on these ideas has become another major interest of mine. The writings of Jung have helped me put this in perspective. My greatest strength and guiding star is intuition, although I am an 'introverted thinking type' in Jung's terminology. This creates difficulties in a culture that has so one sidedly focused on an intellectual approach to problem solving. My ideas about physics are in the spirit of Einstein and I have come to see Einstein's quarrel with his colleagues about quantum mechanics as an almost mythical example of the struggle between intuition and intellect. The title, *Einstein's revenge*, refers to what I foresee as the ultimate victory not only of Einstein's ideas about physics but also of his intuitive approach to science.

The major themes of this book are the development of these ideas on mathematics and physics and the need for our culture to develop intuition the way we have developed intellect. Many of the deepest problems we face today are beyond intellect's ability to deal with, just as a predominantly intellectual approach is unable to see past the existing conceptual framework of quantum mechanics. We must start to reverse the one sided focus on an intellectual approach that is both the greatest strength and greatest weakness of Western culture.

Organization

The great discoveries of this century in mathematics, physics and psychology suggest a new synthesis with implications for mathematics, physics, politics and religion. That possibility is the sub-

ject of Chapter 1. The elaboration of this synthesis is the subject of this book.

By equating existence and consciousness we can apply pure mathematical results (including Gödel's incompleteness theorem) to evolution. Of course, in one sense, there is nothing new here. Evolution is subject to the mathematical laws of physics. However it is not the structure of evolution that is the issue but the immediate experience of evolved creatures. Applying mathematics to this can allow us to get a new perspective on religious values and the creative process. That is the subject of Chapter 2.

The mathematics of creativity and creating mathematics are the subjects of Chapter 3. Reinterpreting the mathematics of the completed infinite as the mathematics of creativity not only allows us to apply this mathematics in new important ways but may also provide the key to extending the logical foundations of mathematics. Appendix A provides a minimal background for this chapter by briefly describing set theory, and giving a proof of Gödel's incompleteness theorem. Appendix C is a first attempt at laying the groundwork for extending mathematics in the way suggested in Chapter 3.

Chapter 4 describes why I think Einstein will ultimately win the argument about God playing dice. Appendix D is a description of a class of mathematical models that may give the victory to Einstein. Appendix B provides a minimal background in physics needed to understand Chapter 4 and Appendix B.

In Chapter 5 I relate mathematics to psychology through Jung's discovery of the archetypes and his ideas about how these relate to number.

If intuition is necessary to see beyond the existing conceptual frameworks in mathematics and physics then it is even more important to apply it to the enormous social problems we face. These cannot have the comparatively simple solutions available in mathematics and physics. Instead they require an ascendancy of intuition throughout our culture. There is no formula for this but one important step is to understand the issues. This is the topic of Chapter 6.

The shadow of a thinking dominated culture is feeling. The shadow nature of feeling is a major contributor to the problems we

face. Intuition is important not simply as a problem solver but also as the mechanism through which we can develop and differentiate feeling. This is the concern of Chapter 7.

A first crude attempt at applying the mathematics of creativity to human institutions is the topic of Chapter 8.

Chapter 9 touches again on religious implications and relates these to what may be a potentially infinite future.

Scope

The scope of this book includes technical issues in mathematics and physics. Most of it should be understandable with the aid of two appendixes (A and B) to anyone with high school level algebra and physics. The more technically involved topics are in appendixes C and D. Appendix A includes a proof of Gödel's incompleteness theorem that is intended to be easily understood by anyone with a little practical experience with computer programming. This appendix starts with a minimal introduction to the mathematics discussed in this book including that needed to understand Gödel's theorem.

Appendix C requires both a knowledge of the C++ programming language and a basic knowledge of set theory such as the minimal introduction in Appendix A. Appendix D require an introductory level understanding of quantum mechanics such as the minimal introduction in Appendix B.

Note on notation

One bit of mathematical notation is needed to read the table of contents or first chapter. ' \equiv ' means logical equivalence. $A \equiv B$ means A and B are the same thing. A number in parenthesis that precedes a reference such as '(123)[1]' is a page reference or, for volumes in the *Collected works of C. G. Jung*, a paragraph reference.

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Chapter 1

A synthesis

This is a scientific and philosophical synthesis of the sort that has been out of favor for some time. It looks at mathematics, physics, philosophy, religion and politics from a single unifying perspective.

There have been attempts to relate the important ideas of this century in quantum mechanics and mathematics to each other and a broader context. Few have suggested we should alter the directions of research in mathematics or physics as a result. That is the what I suggest. I suspect experimental tests of Bell's inequality will lead to results inconsistent with quantum mechanics and this will usher in a revolution in physics as fundamental as quantum mechanics itself. I suggest a speculative class of theories in the tradition of Einstein's approach to quantum mechanics that may account for the existing experimental results and the new results I expect.

Research in the foundations of mathematics has to a degree run up against a brick wall with Gödel's incompleteness theorem. Evolution itself has bypassed those limitations to create the mathematical human mind. We can consciously extend the approach of evolution into an indefinite future. Understanding this aspect of mathematical truth may be a key to understanding how to substantially extend mathematics in the short run. I suspect we are far from exhausting the *existing* mathematical capabilities that evolution has bestowed upon us. I suggest an approach to extending mathematics to more fully realize our potential. The ideas that lead to these approaches to physics and mathematics have broader im-

plications that are best understood in the context of Jung's discoveries about the human psyche.

The cycle of time is ripe for a new conceptual framework to lead to new directions in mathematics and physics and to alter and widen our view of the universe and our place in it. In economic cycles excessive optimism and too much production produces a recession and excessive pessimism. Liberal political dominance shifts to conservatism. Human nature takes whatever works and pushes it until it fails. In the wake the opposite force arises and the cycle continues. This is more than a repetition of the same follies. We learn from the extremes and try to build a middle way avoiding the worst excesses while exploiting the best from both extremes.

Some cycles last for generations. Western civilization has been pursuing an increasingly one sided development of and dependence on a rational approach to problem solving for centuries. This has culminated in a century of the most remarkable scientific and technical progress and the opening of possibilities that were previously unimaginable. It has also lead to the greatest horrors humanity has known and to weapons that threaten the existence of humankind.

In the United States our immense progress in creating material wealth has started to sour. More wealth is concentrated in fewer hands. People are working longer and harder for fewer rewards and with far more anxiety. We are failing to educate many of our children. Civilization is decaying in our inner cities.

Something has gone terribly wrong. We have a myriad of remedies to prescribe. Perhaps our greatest strength is our greatest weakness. We want to figure out how to fix things but maybe it is excessive 'figuring out' that is the source of our problems. Others have suggested that we seem to be suffering something akin to a loss of soul from our one sided reliance on intellect.

The solution is not to abandon intellect or to renounce what we have accomplished. At a deep level there is no solution. The irresolvable conflicts of the soul where different elements strive for dominance in individuals and cultures alike is the engine of human and cultural creativity. To renounce the struggle or grant victory to one side is to end our creative evolution. The cycles never end but they can lead to greater heights or spiral down to despair and

defeat. To stop our downward spiral requires a new ascendancy. Intellect must lessen its grip so that intuition can flower and lead us to explore the other dimensions of our marvelous inner nature.

How do we do this? Through attention. We must learn to listen more carefully to the other voices within. When one voice is loud and clear and others muffled and indistinct this can be difficult. We must learn how to turn a deaf ear to the one who is shouting and listen with attention to the others.

Synthesis and intuition

What has all this to do with a grand synthesis of science philosophy and politics? Intellect analyzes and divides. Intuition sees things as a unified whole. Intellect is like a von Neumann computer and intuition a neural net. The former works with a well defined sequence of steps. The latter recognizes connections, not through a linear process, but by observing the situation as a whole. Synthesis requires intuition both to do and to comprehend. To get beyond the current situation we must begin to understand the marvelous creations of intellect at an intuitive level. We must begin to synthesize these in a way that creates a deeper meaning.

The great advantage of intellect is that it gets results. We may make great intuitive leaps but far more likely than not we fall flat on our face. This puts intuition at a disadvantage. It is why many of the achievements of science required a discipline that constrains intuition and forces it to be the servant of intellect. To develop intuition as a force in its own right as a coequal of intellect and to still discipline it so that what it leads us to can be of practical value is the great problem we now face. For only an ascendancy of intuition can reverse the downward spiral excessive reliance on intellect has begun.

This synthesis aims for practical results. It is not enough to synthesize. One must show why the synthesis is important. Only if intuition, not independent of intellect but playing a leading role, can produce important results will it be able to achieve coequal status with intellect. My intuition has led me to new approaches to the foundations of mathematics (Chapter 3 and Appendix C and

quantum mechanics (Chapter 4 and Appendix D). Ultimately the success or failure of those approaches will do more to vindicate or repudiate the intuition that led to them than any of my arguments.

The ideal

The *ideal* has a prominent role in Western science, culture and religion. We have the Platonic ideal of mathematical truth, culture icons of the ideal life and the religious ideal of God. The *ideal* is a fixed absolute ultimate perhaps infinite goal or destination. Gödel's incompleteness theorem smashed Hilbert's ideal of formalizing mathematical truth. I doubt that any completed infinite totalities exist and thus I doubt that there is any Platonic heaven of absolute mathematical truth. Is the ideal an illusion? The evolution of life seems to be a divergent process with no fixed goals. Evolution *creates* values such as human love. It is not a response to values.

Perhaps we should replace the notion of a final, ultimate or absolute ideal with the understanding that the universe is creatively evolving not to a fixed goal but to an ever more diverse and marvelous range of possibilities. To strive for a fixed ideal is ultimately to stagnate. That is one of the implications of Gödel's theorem that we must apply not just to mathematics but to life itself. Many mathematicians (including Gödel) have continued to cling to the ideal of ultimate mathematical truth existing in an idealized mathematical Platonic heaven of infinite structures. Mathematics is the purest expression of an intellect that wants to figure out the logical steps to obtain a specific goal. It is natural to idealize this process and think in terms of ultimate ideal goals. Up to a point this is necessary and useful. Perhaps intellect pushes it too far.

Intuition does not proceed by a series of steps. Intuition does not have a fixed set of goals. It is always on the look out for interesting relationships. It is always trying to make something out of them. Einstein was perhaps the most deeply intuitive 20th century physicist and perhaps the only prominent physicist for whom intuition and not intellect was the guiding star. He accomplished more than any other physicists of this age even though he spent most

of his professional career futilely searching for a grand synthesis of the revolutions in physics of relativity and quantum mechanics that he did so much to create. He was and is criticized and even ridiculed for this effort and for his refusal to accept that God plays dice. If the class of physical models I propose is at all close to the truth then Einstein will have his revenge with a vengeance.

More than the vindication of his ideas and intuition Einstein's revenge will be a vindication of the power of intuition when it is allowed to be one's guiding star.

Assumptions

The starting point for this synthesis is some simple assumptions.

- There is no special metaphysics that applies to life, human life or consciousness. They are the result of matter structured in a certain way. The self awareness of its own structure *is* (\equiv) the existence of an object.
- Consciousness *is* (\equiv) structured existence. Mathematics, as the study of all logically possible structures, *fully encompasses* the *structure* of life, consciousness and evolution. Pure mathematical results like Gödel's incompleteness theorem have direct implications for human evolution and creativity.
- There are no completed or absolute infinite totalities. Infinite totalities have the property that you can add something to them without changing them. Self aware structures cannot have this property. There may be an infinite collection of finite self aware structures but there is no "completed" infinite. Technically the difference between class and set occurs with the infinite collection of all finite sets. There can be no *structured* infinite objects.

These metaphysical assumptions can be neither proved nor disproved. We justify them in Chapters 2 through 4 and Appendixes C and D. as the simplest assumptions that are consistent with what we understand of the world, ourselves and mathematics. "By their

fruits you shall know them."The assumptions will stand or fall by the results they produce.

Chapter 2

Existence \equiv consciousness

At what moment does human consciousness begin? Perhaps there is no moment. Perhaps the cell consciousness of the sperm and egg merge to a slightly wider consciousness of the growing embryo. Nerve cells grow, connect and start functioning as a brain. Consciousness grows with the complexity and experience of the brain. There is nothing special about the atoms in nerve cells that science is aware of. If we could build a brain from electronic circuits that exactly duplicates the functioning of the human brain would it not be conscious just as a human is? Perhaps even the rocks and soil from which the atoms of the sperm and egg came were also conscious in the sense that they had a direct awareness of their own structure.

What, if anything, could it mean for something to exist if there is no awareness of its existence? Our model of the universe implies that most of what exists is beyond the perception of any conscious human observer. Who experiences what happens in the center of the sun? But what does a conscious observer observe? Is it the blue sky, the excited receptors in the eye, the activity of nerve cells in the brain? It would seem that observation is primarily internal as we can simulate almost any perception by artificial means. We are directly conscious only of the nerve activity inside our body. It is previous experience that allows us to relate this activity to assumed external events. The emergence or disappearance of consciousness are continuous processes and *the same thing* as the development or decay of structures in our bodies. Of course we can

assume there is some metaphysical magic at the moment of conception or birth that breathes a soul into the body. Why should we make such an assumption? It seems that the soul stuff of life is ordinary matter. This is not an affront to the dignity of humanity but a recognition of the divine nature of the dust we come from and return to.

Matter as soul stuff

If matter is conscious soul stuff and human consciousness is just special structures of this soul stuff then everything is made of soul stuff. Everything is conscious of its own structure just as human consciousness is a reflection of the structures of the brain, nervous system and body.

To exist is to be consciousness of ones own existence. The essence and totality of the existence of any thing is its awareness of itself. The self awareness of a rock may be a bit like the awareness of a single point in the visual field if we could somehow isolate an atom of awareness. The rich consciousness we experience is the structured combination of the multitude of atoms of awareness throughout our bodies.

Such metaphysical assumptions are not subject to proof or disproof. They are the simplest assumptions consistent with what we know. Perhaps our individuality is an illusion albeit one of enormous practical value. My consciousness is one small melody in the grand symphony of consciousness. The melody is an integral part of the symphony. If played in isolation it would be something other than what it is. It has a beginning and end but may be repeated with variations over and over. Neither the individual notes no any other aspect of the melody is unique to it. All the elements are repeated in a multitude of ways throughout the symphony. So my life is one unique combination of the elements repeated in all lives. For an immense number of practical reasons I must respect and care for my individuality but in the long run it is the expression of universal themes in that individuality that is important and will endure. The work of life is to evolve those themes and extend them for those to come who will be us. We are not reincarnated in the

sense that our individual memories are passed as a whole to some new individual but we are reincarnated in *every* new baby. Our memories are passed to future generations in the work we leave behind and the lives we touch. Our memories are built from the memories of all those that came before who were us. It is good for the ego to think and feel that this is true just as the soul knows that it is true and shapes our destiny accordingly.

To see ourselves as matter and nothing but matter does not narrow or destroy the human soul. It widens the boundaries of the soul to encompass the entire universe. The continuity of matter and the continuity of consciousness are the same thing. It widens the boundaries in another sense. Our bodies and nervous systems are quite marvelous but they are only the tiniest of beginnings. Gödel's incompleteness theorem shows, as we shall see in the next chapter, that whatever marvelous structures of consciousness evolve they will always be only an infinitesimal fragment of what can be. We are only at the first few vibrations of the first note of a symphony that may unfold with unending complexity throughout an unbounded future.

What exists

What exists? The simplest assumption is nothing. We know that is false. The next simplest assumption is everything. That may be true and in any event can never be shown to be false. In a universe in which there are no completed infinite totalities 'everything' has a well defined mathematical meaning. It is all finite sets. A set is a collection of objects. The first set is the empty set that contains nothing. The next set contains the empty set. It has cardinality 1 since it has one member. The next set contains the empty set and '1'. It has cardinality 2. All mathematical structures can be defined in this way. It is appropriate that they are built from the nothingness of the empty set. Mathematics deals with structure or form but never with substance. Mathematics can tell us a great deal but it can never tell why the experience of the color blue is as it is. Things exist not as structure but as substance. It is the particular substance of our consciousness that is our existence.

That particularness is beyond all explanation and understanding. It simply is. No one can say why the experience of the color blue is at is. This is not a question for which a 'why' answer exists.

Matter is structured and this structure is the structure of consciousness. That all mathematical structures resolve themselves to the empty set is a recognition of the limits of analysis. Consciousness is a unified whole made up of particulars but in no sense is it divisible. Analysis is useful because similar structures correspond to similar experiences. Analysis cannot grasp the ultimate nature of anything. It can only help us see similarities and differences between experiences and thereby help us to shape new experiences. When asking *ultimate* questions like what exists or why is the experience of red like it is we are outside the domain of analysis. Analysis can resolve all structure to the empty set but that is an ultimate explanation of nothing.

When we say that the existence of an object is equivalent to that object being directly conscious of its structure we suggest a dividing line between the object and the universe. We strongly feel that dividing line in our consciousness although we know it is somewhat arbitrary. No absolute dividing line exists. The universe is an indivisible whole and our consciousness merges seamlessly with the universal consciousness. For this whole to have meaning and substance it must be particular and have structure but there are no metaphysical boundaries to this structure. Our consciousness feels individual because of the organization of our bodies and nervous system. That sense of individuality is both an illusion and a practical necessity. It is important for the ego to grow beyond this individuality and to build physical awareness of the seamless whole from which our individuality emerged and will soon return. The expression of this unity is an important theme in many religions. In catholicism it is expressed as the mystical body of Christ which encompasses all humanity and divinity.

What exists is the wholeness and unity of all experience structured in its *necessary* particularness.

The evolution of consciousness

The structure of matter-consciousness evolves both physically and biologically. At first glance Gödel's incompleteness theorem seems to present a problem for the view that human consciousness evolved as a deterministic physical process. How can the human mind understand so much mathematics if it is the result of a process subject to the limitations of Gödel's theorem? There is a deterministic process that can transcend the limits of Gödel's theorem and this is the process through which the human mind has evolved. If one tries to create a single formal system in a deterministic process then one will run head up against the limits of Gödel's theorem. Any single path mechanistic process for enumerating mathematical truth will stagnate at a fixed level in what we might call the hierarchy of mathematical truth. However a divergent process that follows many paths without ever declaring one to be *the* path suffers no such restrictions. There may be *some* path in such a process that ultimately comes to understand and decide any meaningful mathematical question. We may regard the human animal as the highest product of evolution but the rest of nature may not concur.

The 'Gödel hierarchy of mathematical truth' is related to the level of feedback or iteration expressible in a formal system. Mathematically this is expressed as the ordinal number of the formal system but that approach glosses over the combinatorial structure that characterizes the level of feedback. We discuss the detailed structure of this hierarchy in Chapter 3. Higher levels of biological structures correspond to more complex and subtle feedback mechanisms and these in turn *are* higher levels of consciousness. This evolution of structure is a creative process in the deepest sense of that term. Gödel's result insures us that there is no limit to this process and that however much we have obtained it is only an infinitesimal fragment of what is left to obtain.

Of course level of feedback does little to characterize the richness of human consciousness. It is a measure of the potential and limits that are possible at a given evolutionary stage. To understand how these levels may evolve in the future it is instructive to see how they have evolved in the past. There have been major shifts in the central focal point of evolution and each of these

has lead to a new range of possibilities in evolution. At some point the complexity of reproducing molecules in the primal broth was exceeded by the complexity of communities of molecules that reproduced as a unit. Later the complexity of individual cells was exceeded by that of an organism. The result was an array of new possibilities. We seem to be at the earliest stages of the next level. In that level the structure that connect individuals in a society will come to exceed the complexity of the structure of the human brain and nervous system. The focal point of evolution will shift from the individual animal to the society or culture. Of course for that to happen cultures must be able to reproduce. That can only happen through space travel. It will probably involve unmanned probes equipped with all of our cultural knowledge and enough physical and biological material to start civilization from scratch on a distant planet.

The specialization of structures within a cell, organs within a body and individuals in a society each open immense new possibilities. The occurrence of a new level does not obsolete the old. The old is built upon the new and the new continues to coexist independently as an essential thread in the fabric of life. If an attempt at a new level fails the old structures are still there to evolve in other directions or try again. The levels do not form a strict hierarchy. Insect societies are less complex than an individual mammal is. There is no final winner or absolute highest level although those at a particular level may have their personal prejudices. Nature is neutral on the matter and nourishes all who learn how to deal with her effectively.

Each new level requires more resources for an individual. This means less diversity is possible and is a constraint on evolution. There is always a tradeoff between the advantages that come with more complexity of individuals and those that come with more diversity through more individuals.

Evolution beyond earth

Man's mastery of the earth is a unique evolutionary event on this planet. We have multiplied our numbers with techniques never

before possible. This limits the diversity of life. It is a threat to the diversity that created and nourishes us. We must recognize the limits that our finite planet imposes and the possibilities that the seemingly boundless universe opens to us. The instincts to master, control, dominate and overrun the planet must soon be diverted to new dimensions or they will do great harm to us and life on this planet. Finite resources limit freedom and diversity. We instinctively understand that space, as the next but perhaps not *final* frontier, is where we must start looking to divert these instincts. Unfortunately instead of understanding that this is a long term process that we do not yet have the technology to seriously pursue we are exporting our plundering of the planet to our immediate neighborhood in space. Space stations and the manned exploration of space with current technology wastes resources, pollutes space with dangerous debris and does little to prepare the way for a true exploration of space.

In time the complexity of connections between individuals in our society will exceed the complexity of the human brain. The telephone, radio, television and the Internet are important embryonic steps in that direction. In the not too distant future the network connection between human beings will bypass the senses and connect directly to the nervous system. This technology is being developed to help those with impaired senses but it will eventually provide communication capabilities beyond those possible through the senses and that will lead to its use by more than those with impaired sensation. In time our sensations will become collective and global and the most interesting organism will not be the individual person but the new class of individual being that society as a whole has become. The highest level of consciousness will be vested in the collective mind and global network of our society. If that new class of individual, which requires the resources of an entire planet, cannot reproduce and evolve then evolution will stagnate even though we may continue to develop for billions of years on this small planet.

Space is the domain for the reproduction and evolution of this new class of being and technology a necessary mechanism for this new stage in biological evolution.

Evolution beyond the big crunch

The known universe is immense beyond imagination but not beyond calculation. We can give it a size and a lifetime, not with any great confidence in the accuracy, but with good reasons for believing the estimates are correct within an order of magnitude. Of course there is no guarantee and other possibilities such as steady state models cannot be completely ruled out. However the evidence is strong that our universe started as an immense explosion called the big bang and will end in either the thermal death of unending expansion and continuously increasing entropy or will collapse to an immensely dense state known as the big crunch.

Does this mean we live in a huge but finite universe? We will never know with certainty but I prefer to think our universe is potentially infinite and will evolve throughout a future without limits. Historically every previous attempt to bound our universe has been woefully inadequate and I hope the current attempt will suffer the same fate. There is no reason to assume that our *local* big bang is the only big bang in the universe. General relativity requires that universes beyond a certain critical mass and density (black holes) are closed to each other. Things can enter the universe but nothing, not a single bit of information, can leave. If that is absolutely true then it may make little difference if the universe is infinite. However black holes do radiate and do emit information with that radiation. We do not know if that radiation has any correlation with the internal state of the black hole just as we have no idea what happens inside a black hole once the density exceeds certain limits. At that point known physics ends and there are many possibilities.

My favorite fantasy is a potentially infinite universe with an unbounded expanse of big bangs collapsing into big crunches and then exploding again as big bangs. Over time the size of these universes increase. Perhaps the process started with some nearly uniform initial conditions. Just as cells, individuals and cultures are born and die so do entire universes. The big bang/big crunch cycle may be a necessary process to 'recycle' entropy. There is no known process that could decrease entropy but we have no idea what happens in the collapse of a black hole past a certain density.

The enormous densities are a good candidate for creating order out of chaos by some laws completely outside of our current understanding. Some process of this sort is necessary if a potentially infinite universe is to evolve for an unbounded future.

Evolution requires information transfer from those from the big crunch to those in the ensuing big bang who are each other. There are many possibilities. Perhaps there is a way for some small mass to escape the big crunch through interaction with other universes at the points of greatest expansion. Perhaps by altering the shape of the big crunch we can alter evolution in the next big bang. Perhaps there is information from the previous big crunch in some encoded form in our universe that we will someday with yet undreamed of technology be able to decode.

Perhaps the next frontier beyond space is the universes beyond the next big crunch. Perhaps one of the stages of evolution is an entire universe as an individual living, dying, reproducing and evolving.

Chapter 3

Mathematics and creativity

If existence \equiv consciousness then questions about the existence of infinite sets are questions about the possible structure of consciousness. Consciousness is both individual and collective. I see that dot at the end the sentence the paragraph and the monitor that display them all. I am conscious of all this as individual elements and as elements in relation to the whole. A finite structure could be conscious in this sense but not an infinite one. Things can be added to infinite structures without changing them. There is not the same particularness of their elements that exists with finite structures. This suggests that there are *no* infinite structures although the totality of all finite structures may be infinite. I have equated the existence of a structure with the direct experience of that structure. Direct experience is always particular and individual but infinite structures cannot be particular and individual. That is what I mean when I say there are no infinite structures but only infinite collections. The universe may be potentially infinite but all structures and all experience is finite.

In mathematics a class is used to define the universe of all sets. That universe cannot be a set or we fall into a Russell paradox such as the barber who shaves everyone who does not shave himself. If one equates existence with consciousness as we have then the distinction between class and set must occur at the boundary between the finite and infinite. No infinite structures or sets exist there is only the unstructured infinite collections of all finite structures.

What is infinity in mathematics

What does one make of the immense body of mathematics based on infinite structures? The universe may be potentially infinite. Thus questions about whether a Turing Machine (TM) will halt can be meaningful without reference to the completed infinite. (TMs are a special class of computer that have unlimited storage and that can execute any mathematical algorithm with the appropriate program.) This is a question of whether an event will *ever* occur in an indefinite future.

Every initial segment of a potentially infinite universe is potentially empirically accessible to us. Questions about all these segments *may* be meaningful. To say a TM does not halt is meaningful because if it *does* halt we (or our distant ancestors) may eventually know about it.

Most of mathematics that deals with the infinite can be interpreted as dealing with the potentially infinite. For example the question of whether a species will have an infinite chain of descendant species can be defined in a way that requires quantification over the reals. There is no single event that decides this question but it is still meaningful and interesting in a potentially infinite universe. It is determined by a recursively enumerable set of events that can be listed by a computer. Some questions such as the Continuum Hypothesis (see Appendix A.3 on page 81) cannot be. I do not think such questions have any *absolute* meaning. They are questions about formal systems which are within a particular system true, false or undecidable.

To understand in more detail how to develop conventional mathematics along these lines we revive the old idea of the ‘ \cup ’ quantifier. This can replace both the universal and existential quantifiers over the integers: $\cup x F(x)$ is true if and only if $F(x)$ is true for some infinite subset of the integers. This is equivalent to saying that some particular TM has an infinite number of outputs.

This can be generalized to provide an interpretation of the arithmetical sets of integers (those definable with a finite number of quantifiers ranging over the integers). To do so we use the concept of a nondeterministic TM. Here nondeterministic does not refer to probabilistic behavior. A nondeterministic TM is completely de-

terministic in its behavior. However, instead of executing a single program, it executes one or more (up to a recursively enumerable set) programs.

Any arithmetical set of integers can be defined by a succession of a finite number of \cup quantifiers: $\cup x_1 \cup x_2, \dots, \cup x_n F(x_1, x_2, \dots, x_n)$. This in turn is interpretable as some statement about a nondeterministic TM. For two quantifiers, the equivalent statement is that a TM F_t has an infinite number of outputs, such that an infinite subset of these are the Gödel numbers of TMs that in turn have an infinite number of outputs. This is equivalent to a statement about a single nondeterministic TM, since we can easily program such a machine to simulate all the TMs that F_t outputs. This result can be generalized to any finite number of ' \cup ' quantifiers.

Generalizing further one can also define the hyperarithmetical sets of integers as properties of nondeterministic TMs.

We can go a little further. To define the set of all hyperarithmetical sets of integers or equivalently all recursive ordinals requires quantification over the reals. See Appendix A on page 77 if these terms are not familiar. Yet this set also has a natural interpretation in terms of nondeterministic TMs. To show this we introduce a variation on the notion of well foundedness. Program a nondeterministic TM to do the following given the Gödel number of some other TM (F_w) as input. Simulate F_w and for every value that F_w outputs treat this as a the Gödel number of another TM. Simulate this machine just as F_w is simulated, so each of its outputs is interpreted as the Gödel number of another TM. Repeat this process for all outputs of all machines simulated. A path through this tree of simulations is defined to be a sequence of Gödel numbers of TMs. The first element of the sequence is F_w , the second is an output of F_w , the third is the output from the second element in the sequence, the fourth and output from the third etc. F_w is well founded if every path is finite. The set of all recursive ordinals is recursive in the set of well founded TMs. This is not surprising since the definition of well founded TM is close to the definition of a recursive ordinal.

At least this initial fragment of the hierarchy of real numbers has a natural interpretation as properties of TMs.

What is the advantage of analyzing mathematics in this way

and is this mathematics, even interpreted as properties of TMs, relevant to anything at all? Understanding the mathematics as properties of TMs offers the possibility of adding an experimental element to the study of logic. One can write programs of the type just discussed and observe their behavior. This can make it easier to gain insight into this area of mathematics. Something like this has happened. Until recently nonlinear differential equations were, for the most part, ignored as being too complex to understand. Once researchers started playing with simulations, a new branch of mathematics, Chaos theory, was created. Chaos theory is only possible because one can simulate complex systems and observe their behavior.

We know mathematics that requires quantification over infinite sets is useful because it can make it easier to decide practical questions. However, if there are no infinite sets, does this mathematics have a meaning and use in its own right as opposed to being an aid to decide lower level questions? Nondeterministic TM's are similar to the most important process on earth: biological evolution. (At least this is true if biological processes only has access to finite computation.) If we identify a species with a single TM then the question of well foundedness for biological evolution is the question will any species have an infinite chain (the second element of the chain is a descendant of the first, the third a descendant of the second, etc.) of descendant species.

Consider that nondeterministic processes are not subject to the limitations of Gödel's incompleteness theorem. As long as the number of paths of evolution that you follow increases without limit, there is nothing to prevent one particular path from ultimately solving the halting problem for every TM using a chain of mathematical systems each of which proves the consistency of the previous one. This suggests that thinking in terms of nondeterministic processes may be helpful in creating mathematics and that the resulting mathematics is the mathematics of creativity.

Creating mathematics

There are two senses in which one mathematical system can be stronger than another: decidability and definability. Ordinarily the most powerful methods for deciding mathematical questions involve indirect methods operating on abstract structures. However one can artificially construct systems that are weak with respect to definability but strong with respect to decidability.

Zermelo Fraenkel (ZF) set theory contains the axioms that are the most widely accepted foundation for mathematics. The axioms are comparatively simple and have enormous power. See Appendix A on page 77. Virtually all of mathematics is formalizable within this system and virtually all proofs can be done based on these axioms. The power of these axioms come from the complex infinite sets that can be defined with them and the powerful iterative processes that can be defined to operate on these sets.

The power of ZF depends on the large infinite sets or cardinal numbers that can be defined in it. I doubt the existence of the hierarchy of cardinal numbers in ZF. From the Lowenheim Skolem theorem we know that any formal system that has a model has a countable model.

It is possible to construct a formal system that speaks only of recursive processes that would be stronger than ZF in terms of deciding halting problems or the properties of nondeterministic TMs that we have described? The objects in such a system have a concrete existence. We can write computer programs that model them and we can use computer simulations to test our conjectures about their properties. It is conceivable that formal systems constructed in this way might be considered more certain than ZF and still be strong enough to prove the consistency of ZF.

The ordinal numbers are the framework of mathematics. If you define ordinals as infinite sets then you can always construct the next largest ordinal as the union of smaller ones. This is very powerful. It is too powerful. It allows you to wash over the rich combinatorial structure that evolves if you try to develop the same concept but stick to recursive structures. It is a detailed understanding of this recursive structure that will ultimately allow us to go beyond ZF with some confidence that we know what we are talk-

ing about. We are never going to accomplish that with reasoning about large cardinals.

One can give an outline of the functional hierarchy that needs to be developed. Start with a notation for recursive ordinals that allows one to recursively decide the relative size of two ordinals. No fixed notation can have this property for all recursive ordinals so the notation must be expandable. Use this notation as labels in a functional hierarchy. Level 0 in the hierarchy is the integers. Level N contains functionals well founded for objects at level $(N-1)$. Objects at a limit level in the hierarchy are well founded for objects at all lower levels. You need to have explicit recursive labels whose ordering is recursively decidable for the input and output parameters so you know recursively what arguments are legitimate.

You need to generalize these ideas in strong ways while always working with recursive operations on recursive structures. How do you do this? There is a next major level in the hierarchy where the full structure I outlined above is only the successor operation. You can keep repeating this in some sense but how do you do it in strong ways and still always operate on recursive structure? There is of course no general answer to this question.

The ordinals and cardinals in ZF set theory define powerful structures like this *implicitly*. One must construct a candidate notation for recursive ordinals and prove within ZF it has the desired properties. One must then work out the detail of doing and generalizing induction on these structures. Such research is done not with the idea of going beyond ZF but to understand these structures. Mathematicians generally believe that it is too difficult to extend such structures to capture the power of ZF. As long as everyone working in this area believes this it will likely remain a self fulfilling prophecy.

I doubt that we can capture the power of ZF with conventional pencil and paper techniques. The recursive notations for recursive ordinals will become too complex. We need to *implement* the notations on a computer. We need to write programs to carry out iteration and we need to play with these programs trying different approaches to defining and generalizing iteration schemes and ordinal notations. A first approach at doing this is described in Appendix C. Playing the game this way puts less emphasis on in-

tellectual skills and more on intuition.

I believe this process will in the not too distant future lead to systems that are for more powerful than ZF in terms of decidability and that we will have more confidence in than we do in ZF. These systems will not be based on a questionable hierarchy of infinite cardinals. They will be based on computable iteration. Admittedly this iteration will be complex and that will cast some doubt on the consistency of the systems. There is a point where the level of recursive iteration exceeds the capabilities of the human mind to comprehend. That is one of the implications of Gödel's theorem if the mind is a computable process. I think we are far from that point. The axioms of ZF are short and simple. With the right techniques I think we can comprehend the combinatorial iteration techniques implicitly defined by those axioms. In the process I think we will come to understand how to define much more powerful iteration techniques. I believe this understanding will enrich mathematics and have enormously important implications for science and technology.

Alternating between implicit and explicit definability in mathematics is another of the cycles that drive creativity. I am advocating a return to the approach of *Principia Mathematica*[35, 36, 37, 38] where the hierarchy of types was explicitly enumerated. The two improvements I am suggesting are the use of recursive function theory (a theory that did not exist at the time of *Principia Mathematica*) to simplify the construction of the typed hierarchy and the use of computers (which did not exist then either) to get a hold on the complexity of the task.

Typed hierarchies and the Russell paradox

The barber in the small town shaves everyone in town who does not shave himself. Does the barber shave himself? This version of the Russell paradox illustrates the difficulty with having a universal set in a mathematical formalism. Russell felt the solution to this problem was an explicitly typed hierarchy of sets. Functions or operations between sets at one level in the hierarchy could only be defined at a higher level. With Whitehead he formalized a large

body of mathematics in this way in *Principia Mathematica*. Zermelo Fraenkel set theory as described in Appendix A turned out to be a far simpler and more powerful system. It provided a similar typed hierarchy thorough the Axiom of replacement. Today *Principia Mathematica* is primarily of historical interest.

Computers allow us to automate much of the tedious work involved in explicitly typed system like that of in *Principia Mathematica*. Further if we restrict notations and operations to recursive structures we can build models of these systems and play with them on computers to aid our understanding. The driving force for such a system is not to make the hierarchy explicit for its own sake. Rather it is to make the hierarchy explicit enough so that all operations on it can be programmed in a computer and we can thus use computer technology to explore the consequences.

The value in a recursive hierarchy of functional types is in the power of the operations that can be defined. If one only has functions on the integers one can explicitly construct a new function using an old one. Every repetition of this process requires an *explicit* step. If one has functions operating on functions on the integers then one can automate such constructions and diagonalize them. That is one can define a function on functions that generates a new function for any integer N and one can diagonalize this series of functions. The next level is of course are recursive functions that operate on recursive functions on the integers. Of course one can iterate this game in many different ways but the idea of well foundedness captures all the natural iterations that one might think of in a straightforward way. We can have a recursive functional on the integers that has as its range either another functional on the integer or an integer. The output of the functional must indicate whether it is an integer or the representation of another functional. If we have an arbitrary sequence of integers we can apply the first integer to the functional. If that output is a functional we can apply the second integer to it. Again if that output is a functional we can apply the third integer to it. If for every infinite sequence we always get an integer output after a finite number of steps than we say the original functional is well founded. Any recursively specifiable hierarchy functions operating on lower type functions can be specified in this way. At the same time this creates a new class

of object that we can do induction on. We can construct recursive functional that will operate on *any* recursive functional well founded for the integers.

The notion of well foundedness does not follow in a logical way from the lower levels of functional hierarchies that it is generalization of. There is something inherently creative about this concept. Gödel's proof insures that there are infinite number of powerful general concepts that are not in any sense a natural consequence of formalizable laws of mathematics. This underscores the creative nature of mathematics.

Note the difference between well foundedness for recursive integer functionals and the general notion of a well founded set in mathematics. The set theory version is more general. We will generalize integer functional well foundedness to well foundedness for any sequence of recursive functionals that operate on well founded integer functionals. This generalization and others is in a sense already covered by the set theory notion. However the recursive function notion is much richer in structure in a way that has the potential to suggest much more powerful generalizations. The pristine nature of well foundedness in set theory makes it independent of this structure. Of course within set theory one can build the same hierarchy and get the same recursive structures. The point is that you must build the hierarchy to see the richness of this structure and as the hierarchy gets more complex pencil and paper methods will not suffice. You must write computer programs to construct a fragment of the hierarchy as we do in Appendix C.

The mathematics of creativity

If everything that exists is finite, what is the metaphysical status of mathematical questions about a potentially infinite process? Such questions tells us something about the constraints and possibilities in a potentially infinite universe. They can be thought of as the mathematics of creativity for they can help us to determine principles that foster creativity and avoid stagnation. In a sense such questions have an absolute meaning. All the mathematics that I think is absolutely meaningful refers to a recursively enu-

merable collection of finite events. The way in which the truth of those events is related to the truth of the statement does not have any well defined canonical form. Any statements of this nature that seem to be meaningful in a potentially infinite universe are candidates for the mathematics of creativity. Questions like the continuum hypothesis cannot be cast in this form.

We can apply the laws of logic to tell us about processes that continue for ever. That is important if we are concerned about what will happen in an unbounded future. Insofar as we extend our identity beyond the necessary illusion of our individuality to the deeper truth of unity and identity with the creative thrust of all that is, we will have a spiritual interest in an indefinite future. As we shall discuss in Chapter 8 we can begin to understand from mathematics why freedom and diversity are so important to the human species. We may even be able to improve the creativity of the economy by applying the mathematics of creativity.

Understanding higher levels of definability is a creative process that cannot be characterized. However we can characterize decidability relative to some level of definability. In this way we can reason about developing a hierarchy of increasingly more powerful mathematical systems with respect to decidability.

Suppose we wish to set up a mechanistic process that will non-deterministically (following an increasing number of paths) evolve mathematical systems that are increasingly stronger with respect to decidability. We want there to be at least one path that correctly decides (for example) the halting problem for every TM. It is trivial to do this since we can set up a nondeterministic process that will enumerate every initial segment of every real number along some path such that the union of those segments along the path is that real. How can we optimize this process? There are many possibilities. Instead of directly enumerating the statements we are interested in we can assign truth values to a formal system such as first order arithmetic. We can then enumerate the consequences of those assumptions and eliminate any path that is inconsistent. There are tradeoffs between the effort but in enumerating these consequences and the effort put into following more divergent paths. We can ask how do we best allocate resources between these alternatives to maximize the rate at which we precede

along at least one correct path.

This is similar to the tradeoffs between more complex organisms and more descendants that have evolved biologically. It is possible that arguments like the above can help us understand evolution. Biological systems are subject to the tradeoff between diversity and complexity of individuals for similar reasons. The complexity can help an individual better deal with and understand his situation but one can also better respond to ones environment through more diversity. In each case part of the issue is how well one can internally model 'mathematically' aspects of ones environment and situation.

Chapter 4

Einstein's revenge

Godel's incompleteness theorem can only be applied to biology and the human mind if physics can be modeled deterministically. Many physicists feel that this is not true and some have argued that this has implications for the human mind. I think Einstein will ultimately win the argument about whether God plays dice.

In the early years of his career Einstein was the primary instigator of one the great revolutions of 20th century physics, relativity, and a principle instigator of the other, quantum mechanics. Einstein spent much of the rest of his career and life searching for a unified field theory to among other things combine the incompatible revolutions he was largely responsible for. He was much criticized for not following the more certain path of most of his colleges in consolidating and extending the existing theories. In turn Einstein was critical of his colleagues for ignoring or explaining away the conceptual problems in the existing theory. Einstein did not think that God plays dice and felt his colleges gave up far too easily on the admittedly daunting task of finding a more complete deterministic theory to account for quantum mechanical effects.

Of course his colleagues accomplished much during this time and Einstein appeared to accomplish little. Still Einstein's quest may not have been in vain. He may, at the end of his life, see beyond the revolutions he created to the key of providing the more unified and complete theory he sought for so long. This insight was not pleasant for him because it meant that the next great revolution in physics would do to his work what 20th century physics did to

Newtonian mechanics. Still, if his insight proves correct, Einstein will have his revenge with a vengeance. Although both relativity and quantum mechanics will only be approximations to a new and deeper theory, the principles that guided Einstein will emerge triumphant. All the discussion about abandoning classical principles of mathematics and science because of quantum mechanics will be seen as misguided speculation by the unimaginative. There will be no failure of classical logic and mathematics but only a failure of the uninspired to see beyond the conceptual framework of Newtonian mechanics that is still an essential element in quantum mechanics. One cannot even formulate a problem in that theory without first formulating it classically.

Even more than a triumph of these principles this victory to come will represent a triumph of intuition over the one sided intellectual approach to problem solving that has come to dominate Western science and culture. Genius like Einstein's is sometimes portrayed as something that goes so far beyond ordinary intelligence to be incomprehensible. It is certainly incomprehensible to those who want to understand it as an extreme form of intellectual skill because it is nothing like that. It is a completely different mental skill one that our culture does not yet understand, appreciate or know how to develop the way it understands intellectual talent. Perhaps Einstein's genius is not nearly as rare as many think. Perhaps it is our culture's failure to recognize intuitive talent and develop it that makes the highest *expression* of this talent so rare in fields dominated by intellectual approaches to problem solving.

The continuum

I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case *nothing* remains of my entire castle in the air gravitation theory included, [and of] the rest of modern physics(467)[28].

Continuity is among the oldest of mathematical problems. From

a technical standpoint most issues were resolved long ago by defining continuous structures as limiting cases of discrete structures as the elements in the discrete structure increase without limit. However this technical solution does not address let alone resolve the philosophical issues raised by the paradoxes of Zeno. Those question how it is possible to construct a continuous interval from an infinite number of infinitesimal intervals. There is no mathematics to do that. Mathematics based on taking the limit allows us to construct a *discrete* algebra that is the of an unbounded number of discrete systems. It is a discrete algebra because it operates on discrete abstractions like lines, planes, points and circles and not on a *completed* infinite collection of infinitesimal objects. Phrases like “taking the limit” or “passing over to the limit” are misleading. We never reach the limit. All we do is show that certain properties can be made to hold with arbitrarily high accuracy if we can construct a sufficiently large *discrete* system.

The question of whether time, space and everything definable in space-time is discrete is an open one. It is not clear what it would mean to say that time and space are continuous. The things that obey the algebra of continuity are not continuous structures but discrete abstractions. Yet there is nothing abstract about the way physical space-time is built from smaller regions.

Quantum mechanics suggest that the information content of any finite space-time region with finite energy is finite. If true we can fully model physical reality with a discrete model. If this is so it would again be difficult to know what it means to say that space is continuous.

Such philosophical musing may be interesting but the proof is in the pudding. Only if discrete mathematics leads to testable theories at variance with continuous theories will such questions move into the realm of physics. Einstein thought this may happen.

The field concept is central to Einstein’s approach to physics. It suggests we can model physics by looking at what happens at an arbitrarily small region of space. It assumes that there are some simple rules that define how the state evolves in such a region. If we know these rules and the initial conditions we can predict the future with certainty (given sufficient computing resources). The field concept and the locality it implies are fundamental to relativ-

ity. What happens at a point in space is completely determined by the immediate region of that point. Distant objects can only have an effect through the fields they generate and that propagate through space.

Relativity implies continuity. No matter how small a region we select the same rules apply. Quantum mechanics establishes a scale to the universe that defies our ordinary ideas about continuity. As we move to smaller time and distance scales the complexity of what can be observed according to quantum mechanics increases without limit. This alone suggest that at a sufficiently small scale quantum mechanics will fail. There seems to be no way to reconcile these two theories within the existing framework of either. Perhaps it is such considerations that led Einstein to suspect that ultimately physics cannot be based on continuous structures.

The measurement problem

The formulation of quantum mechanics describes the deterministic unitary evolution of a wave function. This wave function is never observed experimentally. The wave function allows us to compute the probability that certain macroscopic events will be observed. There are no events and no mechanism for creating events in the mathematical model. It is this dichotomy between the wave function model and observed macroscopic events that is the source of the interpretation issue in quantum mechanics. In classical physics the mathematical model talks about the things we observe. In quantum mechanics the mathematical model by itself never produces observations. We must *interpret* the wave function in order to relate it to experimental observations.

In 1935 Schrodinger published an essay describing the conceptual problems in quantum mechanics[29]. A brief paragraph in this essay described the cat paradox.

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a

tiny bit of radioactive substance, so small that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The Psi function for the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be *resolved* by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

We know that superposition of possible outcomes must exist simultaneously at a microscopic level because we can observe interference effects from these. We know (at least most of us know) that the cat in the box is dead, alive or dying and not in a smeared out state between the alternatives. When and how does the model of many microscopic possibilities resolve itself into a particular macroscopic state? When and how does the fog bank of microscopic possibilities transform itself to the blurred picture we have of a definite macroscopic state. That is the measurement problem and Schrodinger's cat is a simple and elegant explanation of that problem.

It is important to understand that this is not simply a philosophical question or a rhetorical debate. In quantum mechanics one often must model systems as the superposition of two or more possible outcomes. Superpositions can produce interference effects and thus are experimentally distinguishable from mixed states.

How does a superposition of different possibilities resolve itself into some particular observation? This question (also known as the measurement problem) affects how we analyze some experiments such as tests of Bell's inequality.

So far there is no evidence that it makes any difference. The wave function evolves in such a way that there are no observable effects from macroscopic superpositions. It is only superposition of different possibilities at the microscopic level that leads to experimentally detectable interference effects.

Thus it would seem that there is no criteria for objective events and perhaps no need for such a criteria. However there is at least one small fly in the ointment. In analyzing a test of Bell's inequality (as described in the next section) one must make some determination as to when an observation was complete, i.e. could not be reversed. These experiments depend on the timing of macroscopic events. The natural assumption is to use classical thermodynamics to compute the probability that a macroscopic event can be reversed. This however implies that there is some *objective* process that produces the *particular* observation. Since no such objective process exists in current models this suggests that quantum mechanics is an incomplete theory.

Bell's inequality

In 1935 von Neumann published a proof that no more complete theory could reproduce the predictions of quantum mechanics[34]. This proof was accepted for nearly three decades even though Bohm published an example of a more complete theory in 1952[6]. Bohm thought at the time that his theory at some level must disagree with standard quantum mechanics and thus was not in conflict with von Neumann's proof.

Bell suspected there was something wrong with von Neumann's proof and, in the early sixties, he wrote a refutation[4]. At the end of this paper (which was not published for several years) he suggests that the broad result claimed by von Neumann was not possible. Influenced by Bohm's work he wondered if one might show that no local realistic theory could be consistent with quan-

tum mechanics. The term *realistic* does not have an unambiguous meaning in physics but Bell defined what he meant. In 1964 he published a paper[3] that described an inequality (now referred to as Bell's inequality) that any local theory must obey. The equality relates two experimentally controllable parameters to two detections. Figure D.1 on page 126 illustrates the setup. In this figure the experimental parameters are referred to as polarizers. The inequality relates the angles between the polarizer to the probability that there will be a joint detection of two events. In a local theory in which the measurements are determined by an objective state (local realistic theory as defined by Bell) there are various inequalities that constraint this correlation function. Quantum mechanics predicts these constraints do not hold. The inequality is only predicted to hold if a change in the probability of joint detections occurs in less time that it takes light to travel from either polarizer to the more distant detector. Quantum mechanics predicts this change can happen in an arbitrarily short time.

In the 1970's Eberhard derived Bell's result without reference to 'realistic' theories[12, 13]. It applies to all local theories. Eberhard also showed that the nonlocal effects that quantum mechanics predicts cannot be used for superluminal communication.

How does quantum mechanics violate locality? Two principles of physics are involved: the singlet state and and the act of measurement. In quantum mechanics the properties of pairs of particles in a singlet state remain connected even if the particles become separated by a great distance. They are still part of a single wave function. The wave function is not something that exists in physical space. It is defined only in configuration space where there are a separate set of spatial coordinates for every particle. The wave function evolves locally in configuration space. To make predictions in physical space we must project the configuration wave function model onto physical space to compute the probability that an event will be observed at a detector. The combination of wave function evolution in configuration space and this projection operation is irreducibly nonlocal.

Because quantum mechanics claims that probabilities are irreducible it is not possible to send a superluminal signal. The only way to model what is happening mathematically is for information

to be transferred superluminally from one of the polarizers to the detector more distant from it. However because of quantum uncertainty there is no way to know in what direction this information transfer occurs. You can only prove there must have been the superluminal transfer of information by comparing the results from the distant detectors. Superluminal transfer of information without superluminal signals is only possible in a theory that claims probabilities are irreducible.

Physicists often claim that there is no superluminal transfer of information predicted. They claim that something is happening that does not fall within the domain of our *classical* mathematics. Perhaps this is true but I prefer to write as if classical mathematics holds even for quantum mechanics. There is not a shred or experimental evidence that suggests it does not. There have been experimental tests but none of them are conclusive. The experimental verdict is still out on locality in nature.

Bell's result has, to a degree, converted the metaphysical measurement problem to an experimental question. If locality holds then there is a space time structure to the changes in the wave function associated with an observation and through tests of Bell's inequality we will be able to experimentally observe this structure which is outside of any accepted physical theory.

Is quantum mechanics complete?

Einstein did not believe that God plays dice and thought a more complete theory would predict the actual outcome of experiments. He argued[15] that quantities that are conserved absolutely (such as momentum or energy) must correspond to some objective element of physical reality. Because quantum mechanics does not model this he felt it must be incomplete.

It is possible that events are the result of objective physical processes that we do not yet understand. These processes may determine the actual outcome of experiments and not just their probabilities. Certainly that is the natural assumption to make. Any one who does not understand quantum mechanics and many who have only a superficial understanding naturally think that observations

come about from some objective physical process even if they think we can only predict probabilities.

There have been numerous attempts to develop such alternatives. These are often referred to as 'hidden variables' theories. Bell proved that such theories cannot deal with quantum entanglement without introducing explicitly nonlocal mechanisms[3]. Quantum entanglement refers to the way observations of two particles are correlated after the particles interact. It comes about because the conservation laws are exact but most observations are probabilistic. Nonlocal operations in hidden variables theories might not seem such a drawback since quantum mechanics itself must use explicit nonlocal mechanism to deal with entanglement. However in quantum mechanics the non-locality is in a wave function which most do not consider to be a physical entity. This makes the non-locality less offensive or at least easier to rationalize away.

It might seem that the tables have been turned on Einstein. The very argument he used to show quantum mechanics must be incomplete requires that hidden variables models have explicit nonlocal operations. However it is experiments and not theoretical arguments that now must decide the issue. Although all experiments to date have produced results consistent with the predictions of quantum mechanics, there is general agreement that the existing experiments are inconclusive[26]. There is no conclusive experimental confirmation of the nonlocal predictions of quantum mechanics. If these experiments eventually confirm locality and not quantum mechanics Einstein will be largely vindicated for exactly the reasons he gave. Final vindication will depend on the development of a more complete theory.

Most physicists (including Bell before his untimely death) believe quantum mechanics is correct in predicting locality is violated. Why do they have so much more faith in the strange formalism of quantum mechanics than in basic principles like locality or the notion that observations are produced by objective processes? I think the reason may be that they are viewing these problems in the wrong conceptual framework. The term 'hidden variables' suggests a theory of classical like particles with additional hidden variables. However quantum entanglement and the behavior of multi-particle systems strongly suggests that whatever underlies

quantum effects it is nothing like classical particles. If that is so than any attempt to develop a more complete theory in this framework can only lead to frustration and failure. The fault may not be in classical principles like locality or determinism. Their failure may only be in the imagination of those who are convinced that no more complete theory is possible.

One alternative to classical particles is to think of observations as focal points in state space of nonlinear transformations of the wave function. Attractors in Chaos theory provide one model of processes like this. Perhaps there is an objective physical wave function and quantum mechanics only models the average or statistical behavior of this wave function. Perhaps the amplitude of this physical wave function determines the probability that the wave function will transform nonlinearly at a particular location. If this is so than probability in quantum mechanics combines two very different kinds of probabilities. The first is the probability associated with our state of ignorance about the detailed behavior of the physical wave function. The second is the probability that the physical wave function will transform with a particular focal point.

A model of this type might be able to explain existing experimental results and still never violate locality. In Appendix D I describe a class of models of this type based on using a discretized finite difference equation rather than a continuous differential equation to model the wave function. The nonlinearity that must be introduced to discretize the difference equation is a source of chaotic like behavior. In this model the enforcement of the conservation laws comes about through a process of converging to a stable state. Information that enforces these laws is stored holographic like over a wide region.

Most would agree that the best solution to the measurement problem would be a more complete theory. Where people part company is in their belief in whether such a thing is possible. All attempts to prove it impossible (starting with von Neumann[34]) have been shown to be flawed[4]. It is in part Bell's analysis of these proofs that led to his proof about locality in quantum mechanics. Bell has transformed a significant part of this issue to one experimenters can address. If nature violates locality in the way quantum mechanics predicts than a local deterministic theory

of the kind Einstein was searching for is not possible. If quantum mechanics is incorrect in making these predictions then a more accurate and more complete theory is a necessity. Such a theory is quite likely to account for events by an objective physical process.

Chapter 5

Number as archetype

The metaphysical assumptions in Chapter 1 allow us to connect results in mathematics and physics directly to the human mind and psyche. The physical structure of the mind and its conscious structure are the *same* thing. Jung felt that there was a unity of matter and psyche related to the archetypal structures in the psyche and to number. The physical structure of our bodies was determined by evolution. Some structures were evolved earlier and others later. More complex structures are built on simpler ones. Jung recognized that the same is true of our minds. Archetypes are the psychic shapers of ideas, images and behavior that have been molded by evolution over the eons. Archetypes are not emotions, thoughts or mental images. They can be sources of all of these. Archetypes do not have a specific meaning. They create meaning.

After C. G. Jung had completed his work on synchronicity in “Synchronicity: An Acausal Connecting Principle,” he hazarded the conjecture, already briefly suggested in his paper, that it might be possible to take a further step into the realization of the unity of psyche and matter through research into the archetypes of the natural numbers. He even began to note down some of the mathematical characteristics of the first five integers on a slip of paper. But, about two years before his death, he handed the slip over to me with the words: “I am too old to be

able to write this now, so I hand it over to you.” — Marie-Louise von Franz, from the preface of *Number and Time*[32].

Synchronicity

Jung’s idea of Synchronicity as an acausal connecting principle somewhat misses the mark. Meaning is created out of the causal, chaotic events of life. For example, human love comes in part from the evolutionary advantage of parents caring for their offspring. It evolved from a seemingly mechanistic processes and yet it is one of the most profoundly meaningful experiences in all our lives. One might say that the causal relationships that give rise to these feelings are one dimension or connecting principle and the experience itself is another dimension or connecting principle. I think they are one in the same. That is what I mean by: existence \equiv consciousness. There are not connections related to meaning and connections related to cause there is only experience in its structured wholeness.

Jung misses the mark more than a little when he sees Synchronicity in extra sensory perception experiments like those of Rhine[23]. Carefully controlled experiments of this nature always seem to yield negative results. Of course human intuition is capable of remarkable achievements but literally transcending time and space to gain knowledge of distant events that could not have been gained in any other way is not among the capabilities of intuition or any other part of the human mind and psyche. The unity of matter and psyche or *unus mundus* that Jung saw parapsychology as evidence for is simpler and more direct than he thought.

Marie-Louise von Franz writes beautifully of myth and fairy tales. Unfortunately mathematics in contrast to fairy tales has a definite and unique interpretation. To write of mathematics as one does of myth is to violate the nature of mathematics. *Number and Time* is not a good book.

The unity of psyche and matter

Jung may have missed the mark in the details but the intuitive vision that he was trying to get at is correct and profoundly important. Numbers or more precisely the mathematical properties of numbers are the key to understanding the *unity* of psyche and matter. Jung thought numbers were discovered and thus similar to archetypes.

The mathematical properties of numbers are discovered and absolute and yet creative. At some point in time each of them may be discovered but at any point in time only an infinitesimal fragment of them can be known. These seemingly paradoxical properties come from mathematics' concern with the potentially infinite in a universe in which everything that exists is a particular finite experience. This is one clue to understanding the relationship between the particular and the infinite.

There are levels of understanding. Understanding the mechanisms of some process is sometimes thought of as full or complete understanding. It takes little thought to recognize how mistaken this is. One can easily learn the rules (or mechanisms) of the game of chess. The limitations of this mechanistic level of understanding become obvious as soon as one plays an opponent with more experience. For understanding to be of practical value it almost always must go beyond mechanisms. It must predict elements of behavior without fully modeling the mechanistic evolution of a system. This is true even in something as combinatorially simple as a game of chess.

The archetypes have been built from an unfathomable history of experience. The details of those experiences are different but there are structural similarities that are universal enough to find their way into our genes. The generality that makes these experiences important enough to incorporate in our genetics makes it problematic to apply the experience to specific situations. To a large degree life is a process of refining archetypal material into ideas, intuitions, art and behavior that have value in our life and times. Jung saw medieval alchemy as providing both a powerful metaphor for this process and as an intuitive and intellectual study of the process albeit one tinged in superstition[22, 21, 24]. Archetypal ma-

terial related to sex, birth and family is among the most basic and direct. Even in this domain the refinement of archetypal material is far from straightforward as the immense problems we are having today in family structure confirm. Much of the difficulty is rooted in the contradictory and competitive nature of the archetypes.

The problem is not just to refine archetype images individually to golden nuggets of practical value. Their deepest values can only be realized through a union of contradictory claims, Jung's *Mysterium*. Because of the immense time scale over which the archetypes developed they are inevitably concerned most fundamentally with the creative aspects of life. It is those aspects that have universal meaning and value. Creativity is ultimately incompatible with any single path of development. It can only be rooted in contradictory forces with no ultimate resolution but only provisional resolutions at a single point in a divergent evolutionary process.

The finite and the infinite

We can gain insight into the archetypes through the finite and infinite in mathematics. The Turing Machine (TM) is the most elementary example of the relationship between the finite and infinite. A TM will either halt or not halt. This is an absolute fact. There is no general method to *decide* the question for each TM yet for each TM there is a mathematical principle that will decide the question for that TM. As TMs become more complex new undiscovered mathematics must be understood. There is no finite way to encompass mathematical truth even when that truth is constrained to statements about the future states of a computer following the precise deterministic steps of a program. This is important not only for questions that refer to an indefinite future. It is rare that one can model a system in complete detail. Most useful mathematics involves that ability to decide things about a system with less than perfect knowledge of its state. Often such mathematics falls in the same category of the halting problem with respect to decidability. It is based on general mathematical properties of a system and the implications of those properties from mathematics.

We must create meaning to broaden our understanding of abso-

lutely true mathematics. The idea of well foundedness for all paths encoded by a real number (as discussed in Chapter 3 on page 19) is a powerful mathematical concept. It provides strong extensions of elementary induction. Is this concept created or discovered? There are not and cannot be real numbers or well founded structures in our finite particular experience. Yet the concepts are relevant to a potentially infinite universe. Evolution creates the brain structures that in turn create these concepts. They evolve because of the advantage of building models of reality that can be extrapolated into the future with logic.

The unknowable creative aspect of the properties of numbers and the unknowable creative aspect of matter are the same thing. It is this creativity that has expressed itself in our world as it is today and that continually unfolds in ways that we can never predict or control. The archetypal images of the human psyche are formed from this creative process and point towards it.

The mathematics of creativity as described in Chapter 3 allow us to know with mathematical precision some of the properties and constraints of creativity. It allows us to make connections between some human instincts and general mathematical properties. It opens the Jungian notion of archetype to mathematical analysis. This does not lessen the divine mystical nature of archetype. On the contrary it shows the divine mystical nature of mathematics.

The necessity of archetypes

How does evolution deal with the mathematical constraints on an organisms ability to control or predict its environment? With its seemingly simple goal of , evolution must be concerned about such questions. Survival can be enhanced by understanding the world. That takes resources for sensation and resources for building and extrapolating models of the world. These in turn fully encompass the problems of deciding mathematical truth.

There is no optimum solution. One should expect many, approaches and many tradeoffs with complex feedback mechanisms between them. It is important for the fit to survive but is also important that the survivors do not become too narrow in their ap-

proach to survival. What may be the best approach in one set of circumstances may be a disaster as things change. This requires instincts for many different approaches and a decision algorithm to select between them depending on circumstances. No selection is ever certain to be correct. Thus decisions are tentative and the path not chosen still clamors for attention.

One can begin to understand at the level of mathematics why the human psyche must be so diverse and seemingly chaotic. All the approaches to patterns in life that have been built up over the eons are there. It is primarily intuition that first recognizes a match and brings something to consciousness. Both intellect and feeling are essential in evaluating the content both in terms of seeing if it applies to the current situation and understanding how to use it. This process is concerned with individual survival but it also concerned with survival in the broadest sense. Thus some of the images and ideas may be harmful to the individual but helpful in a broader sense.

The psychic structures that motivate and inspire us cannot be characterized in any simple way any more than mathematical truth can be. They do not have well defined goals because they are concerned with creative evolution. The struggle between the elements of the psyche is necessary. There is no way to decide what is to be done in a given situation. The different possible approaches must all be given a hearing. Then one decides based on an individual approach to life. Of course this is not a free decision. If one consistently neglects a major component of ones being, that component will make itself felt. Some situations are so important and so directly connect to archetypal material that the decision is made for you. The decision is largely independent of the individual path one has chosen.

Chapter 6

The ascendancy of intuition

In previous chapters I have suggested that intuition in a leading role is essential to see beyond the existing conceptual frameworks of mathematics and physics. This does not mean that intuition is unused or unimportant in those fields now. However I have found it impossible to develop my ideas in the framework of conventional institutions. The barriers for someone whose greatest strength and guiding star is their intuition are formidable. To succeed you must produce results that are of immediate and direct use in an intellectual approach. That is the primary criteria on which you are judged. You must use intuition as the servant of intellect. If you let it lead the way you become lost in problems and ideas that cannot readily be translated to a form that intellect can use. The dominance of intellect and secondary role for intuition was a necessary stage in development but we are at (actually long past) the point when that dominance is necessary or desirable. Intellect will not readily release its grip. To understand a little of the psycho-dynamics involved we need some understanding of Jung's typology.

Psychological types

If there were some formula for success, or for predicting the future then evolution could proceed along a single path of development converging to this optimal solution. Since this is not possible, di-

vergent approaches are needed. This requires diversity at all level of structure, between species, between individuals within a species and within an individual. Jung's *Psychological Types*[25] characterizes some of this diversity both between individuals and within an individual.

Jung saw universal 'types' in human personality. These types are all present in all of us but there tends to be one predominant type or normal mode of organizing our experience. The types are both complementary and competitive. One can gain insight into oneself and others by understanding the structure that Jung described but one must not interpret it too narrowly or literally. The reality that underlies this simple intellectual model is far more complex and problematic than any description of it can suggest. I urge you to read Jung, but I will give a brief summary to provide context for my remarks.

Jung begins his description by noting two approaches to evolutionary success. One can have many offspring with few defenses and a short life or one can, at the expense of lower fertility, invest more in the individual equipping each with more defenses and a longer life. This fundamental tradeoff can appear in many forms. Some individuals limit their activities and carry them on intensively. They are inner directed or introverts. Others are extensive in their activities and of necessity less intense. They are extraverts. We live in a strongly extraverted culture and thus introversion tends to be viewed in inferior terms and seen almost as a defect. Jung, who considered himself an introvert, did not see it that way. Introversion and extraversion are *attitude-types*.

The attitude-types ... are distinguished by their attitude to the object. The introvert's attitude is an abstracting one; at bottom, he is always intent on withdrawing libido from the object, as though he had to prevent the object from gaining power over him. The extravert, on the contrary, has a positive relation to the object. He affirms its importance to such an extent that his subjective attitude is constantly related to and oriented by the object(557)[25].

In contrast to the attitude-types Jung defined the *function-types*.

These refer to the predominant mode of processing information and the orientation of that mode. The rational types process information somewhat like a von Neumann computer. They organize experience in a framework of cause and effect. The irrational types process information somewhat like a neural net. They organize experience in a framework of patterns with more complex and higher dimensional structure than the linear processing of a von Neumann computer.

Thinking uses rational processes, and its own laws or models, to bring elements of both internal and external experience into conceptual connection with one another. *Feeling* uses rational processes to recognize the *value* of an experience or situation. Thinking relates experience to a conceptual framework. Feeling relates experience to a framework of what is valuable or important. The two functions may be in accord. For example, if there is a physical threat. Understanding how to avoid this is what is important. In dealing with other people these are often in conflict. The truth often hurts and what one thinks the truth is hurts even more often. Everyone's conceptual framework, in so far as they have one, is different. Statements that will enhance the feeling situation are often not in accord with one's own conceptual framework and vice versa. Saying what you believe to be objectively correct can get you in a lot of trouble or alienate people you care about.

At a superficial level thinking and feeling types can be mutually attractive and compatible. The feeling type reacts to enhance the feeling situation and thus it seems to the thinking type that they have compatible conceptual frameworks. The feeling type is able to move the situation to what is of value because the thinking type is largely unaware of how these movements are being controlled by the feeling type. This superficial attraction and compatibility can lead to a fundamental impasse if too deep a relationship is attempted. The superficial compatibility comes from the inherent differences that gives each their own sphere of influence. If either tries to move out of their sphere the fundamental difference becomes apparent.

The sensation types are oriented by the patterns they recognize in internal or external experience. The intuitive types are oriented by patterns that indicate where a situation came from or where it

is leading to. "In intuition a content presents itself whole and complete, without our being able to explain or discover how this content came into existence(770)[25]." The same is true of sensation. When we recognize our friend's face we cannot say what steps we went through to do this. Intuition and sensation are pattern recognition processes. The difference is that sensation focuses on the content of an internal or external experience. Intuition focuses on the precursors of an experience or where an experience might lead. We cannot explain pattern recognition the way we can explain a rational process. That does not mean it is beyond rational or causal explanation as Jung seemed to think. We can describe how a neural net comes to recognize a pattern. We can break this up into causal steps although these are nothing like the causal steps in a rational deduction and do not explain the process the way the steps in a rational process explain the conclusion.

The rational and irrational types of Jung bear a stinking resemblance to the comparatively recent left brain and right brain discoveries in biology. The parent, adult and child of Eric Berne[5] are another greatly simplified way of organizing the same material as it applies to the extravert. Adult corresponds to thinking, parent to feeling and child to intuition and sensation.

We all have all of these capabilities. We have different strengths and weaknesses. We develop and differentiate them to different degrees. We orient ourselves and our experience in different ways.

To the degree that we one sidedly develop one of these attitudes and functions in our consciousness there will be a compensating effect from our unconscious. The function types form pairs of opposites. Thinking is opposed to feeling and sensation is opposed to intuition. Of course we can think about both the value of an action and its objective meaning. However feeling is not thinking about value. Feeling is *organizing* experience from the standpoint of feeling. A Star Trek episode illustrates this in a way that puts a very negative light on it as one would expect in a thinking dominated culture. Captain Picard is being tortured by an alien that wants to break him. He is shown five lights but told that there are only four and asked how many lights he sees. Each time he answers five he experiences intense pain. He never gives in but when he discusses the incident later he confesses that at the end he saw only four

lights.

The best and the brightest

One can get some perspective on the dominance of extraverted thinking and the problems this leads to from the phrase 'the best and the brightest' used to describe the leaders that determined policy during the Vietnam war. They may have been the brightest but they were hardly the best! They had too narrow an understanding of the issues. They were brilliant at intellectual problems but the most formidable issues they faced were far beyond any intellectual solution. Millions of Americans had an intuitively deeper understanding of that conflict than the brilliant leaders who were responsible for so much tragedy.

Robert McNamara, a major architect of our Vietnam policy, has recently acknowledged the mistakes, but still speaks of it in terms of the threat that was felt from communism[27]. He explains the history of Germany before the Second World War and the costs that were involved in appeasement. He still does not get it. The lesson of Hitler is the lesson from the First World War and the punitive treaty that followed. The more insightful at the time understood that this was not a peace treaty but a twenty year truce. In treating a defeated Germany as a nation that deserved no consideration the allies created the fertile ground for a Hitler to thrive. It is a *moral* failure to see your enemy is evil incarnate. If you cannot come to terms with the shadow side of your own personality or nation then you project that shadow onto others amplifying their faults and blinding you to your own. It is especially difficult for the intellect to deal with the shadow. Intellect wants clear cut distinctions and those are not possible. The failure of the treaty of Versailles and of Vietnam are the same moral failure of projecting one's shadow rather than dealing with it. One sided dominance of intellect is a major contributing factor.

Before we can go forward we must go backwards. We must look to the methods and disciplines from a time when development of human potential was not so badly skewed. We must pick up the threads that were abandoned and relegated to an underground

status in our headlong rush to pursue the fruits of intellect.

Astrology and intuition

There is a popular interest in astrology because of the superstition associated with it. But that is not its only attraction. Jung has pointed out that astrology is a discipline for developing intuition. This is the primary reason serious people seriously practice it. Astrology is useless as an intellectual model describing correlations between stellar positions and personalities. That however is irrelevant. Serious successful practitioners do not use astrology as an intellectual model. They use it as a backdrop and aid to their intuition. Used it that way it works for them. If Western culture had done more to develop intuition there would be no need to revert to ancient methods that are still immersed in superstition.

Intuition as pattern recognition

Intuitive insights suddenly appear as ideas or feelings. The solution to a problem leaps into your mind. You have a strong anxiety about taking some action but cannot understand why. Where do these things come from? You are continually, without conscious attention, recognizing patterns in the stream of sensations that impinge upon you. If one of these is important it is suddenly brought to your conscious attention. Perhaps you are playing a computer game and you suddenly recognize the voice of the company president coming down the hall. Even though you are strongly focused on the game, the recognition of that voice destroys your concentration as it is brought to conscious attention.

This same pattern recognition process is happening in another way. Your experiences as an unfolding sequence in time are continually being compared to previous sequences to see if there is some important relationship. If one of these is important enough it is brought to conscious attention as an idea or feeling. With sensation what is brought to consciousness is a recognition of something in the environment. With intuition what is being recognized

may be unclear. Sensation mostly recognizes things we have experienced before although it does respond to images that are more fundamental. For example in a child entering adolescence a sexually attractive body stirs feelings that have deeper origins than life experience. In youth the recognition of a pattern in such images leads to feelings and then experience. The recognition does not have an origin in experience. It is not personal. It has an archetypal character.

With intuition far more of the search for patterns is concerned with archetypal rather than personal experience. Little in our lives is fundamentally original. Almost every situation we encounter is similar to an immense number of previous situations. These similarities are not limited to the human species. They go back through the history of evolution. For example walking past a dog that feels you are violating its territory raises instincts and actions in the dog that are not so far removed from similar human instincts. Evolution molds life to respond to recurring situations. The *I Ching*[39] is a catalog of recurring life patterns. It describes them so they can be directly understood by consciousness. That is part of the value of the *I Ching* as an intuitive discipline. It strengthens our conscious understanding of the patterns that intuition recognizes. With a better conscious understanding we know more about what to make of these patterns and we can better focus our intuition as a result.

There is no way to say why a neural net produces one response rather than another. You can do a detailed analysis of the state and explain exactly why this history and input produces this response but that is no explanation. Because intuition is a generalized pattern recognition process you cannot break up the result into a series of steps or analyze the process for mistakes. The way you discipline and develop intuition is completely different than the way you develop intellect. You train a neural net by exposing it to a variety of inputs and reinforce those responses that are correct. This is fine when we have an objective criteria for correctness. We need to develop intuition in domains where do not yet have an objective criteria for correctness.

The *I Ching* and astrology are traditional methods of developing intuition about psychology, sociology business and politics. These

methods have stagnated and assumed an underground status. To some degree this was a practical necessity. Intuition leaps far ahead of intellect and intellect cannot fill in the vast canyons that intuition crosses. These leaps can miss the mark. Intuition, left to its own devices, soon finds itself trapped at the bottom of a canyon. Einstein's quarrel with many of his colleagues is an almost mythical example of this struggle. Einstein leaped into a canyon from which he could not escape.

Einstein's intuition

It is worth looking closer at this quarrel. Extraverted thinking that dominates our culture including science draws its energy from the external facts or experimental results. Quantum mechanics is extraordinarily successful at explaining those facts. The refinements that his colleagues made to the theory while Einstein was pursuing his futile quest for a more complete theory have made quantum mechanics, and specifically quantum field theory the most accurate theory man has ever developed by a wide margin. Certainly his colleagues had reason to complain when they accomplished so much and Einstein so little. Einstein respected the enormous achievement but felt we must start over.

There is no doubt that quantum mechanics has seized hold of a beautiful element of truth and that it will be a touchstone for a future theoretical basis in that it must be deducible as a limiting case from that basis, just as electrostatics is deducible from the Maxwell equations of the electromagnetic field or as thermodynamics is deducible from statistical mechanics. I do not believe that quantum mechanics will be the starting point in the search for this basis, just as one cannot arrive at the foundations of mechanics from thermodynamics or statistical mechanics(461)[14].

We must start over because you cannot derive a causal theory from a statistical one. Einstein had an inner vision or intuition

about what was and was not a good fundamental theory. A theory that did not match that inner vision was sadly lacking no matter how successful it became. Quantum mechanics did not match this vision and no amount of doctoring it to cover a wider range of effects or achieve greater accuracy could help. Quantum field theory, which combines special relativity and quantum mechanics, was anathema to him.

Einstein never had a good word for the relativity version of quantum mechanics known as quantum field theory. Its successes did not impress him. Once in 1912, he said of the quantum theory the more successful it is, the sillier it looks(24)[28].

His colleagues impressed by the enormous success of quantum mechanics did not share his view. They understood how the theory fell short of what had been accepted principles for a physical theory. Their solution was to modify these principles. Thus we have a host of interpretations of quantum mechanics each with its own special metaphysics and new principles for a fundamental theory. For the extravert the idea must succumb to the data. For the introvert it is the opposite. Neither principle works universally. That is why an opposition is needed.

Why do I insist that the idea will ultimately win out in this contest? It is the accumulation of intuitive problems with the theory. They are what make the theory look sillier the more successful it becomes. The problems are listed in Appendix B. Beyond this intuition is able to consider possibilities that intellect cannot deal with. Intuition is always ready to start over. Intellect is loathe to do so because without its existing conceptual framework it is lost it has nothing to orient itself with.

For intellect to proceed in physics it must have or work out the the mathematics in some detail. Intuition can play with ideas at a looser level. Intuition can leave the conceptual framework of classical particles that quantum mechanics is trapped in. Without knowing the details it can match patterns and see where connections are possible in a different framework. Of course this process is far more error prone than a more narrow intellectual approach,

but for many problems it is the only possible approach. It is the starting point that must precede an intellectual solution.

Developing intuition

How can we develop intuition, let it lead the way and yet hold it back from leaping into the abyss as Einstein did? Of course I do not think that Einstein was wrong but in his lifetime he was not able to accomplish what he intended. We can afford to support a few Einsteins without practical results, but for intuition to become more universal it must become more developed and differentiated. We must know when and how to use it and we must know with some, albeit imperfect, reliability when it leads us to far afield from what is practically possible.

The one sided culture I am so critical of has provided one important tool for this. The computer allows us to create artificial universes play with them and with our ideas so that we can discipline and refine our intuition without making it the servant of intellect. I can learn complex technical material best if I can program it and play with the program. A mass of equations without the opportunity to make them alive in a computer is virtually meaningless to me. It is not that I am unable to understand them but the mode that I can understand them has to involve an element of playfulness and has to be tolerant of many silly errors which I continuously make. Although a computer is completely intolerant of mistakes it allows as many tries as you are willing to make to get it right.

Intuition is not as quick as intellect but it is deeper. Intellect can easily grasp things as a series of complex operations. This is impossible for intuition. Intuition must know how the operations relate to each other and to a host of similar operations that are already understood. This takes time and it takes playing with ideas. For complex systems this is impossible without a computer to handle the details. Of course there is no intuitive only or intellectual only learning. All learning involves sequences of steps, playing with ideas and relating new ideas to old ones. The difference is one of emphasis and dominance.

The computer combined with communication technology is a powerful aid to intuition in another way. It can create learning and dialogue networks of people concerned about a particular issue. The misnamed newsgroups on Internet serve this purpose. Although they do contain some news the vast majority of traffic involves networks of people exchanging ideas and learning from each other material that is far from new. For me this was an effective way to learn the language and some of the technical content of quantum mechanics. It helped me to extend my ideas and put them in a context that others could more easily understand.

Technology can change the value of human talents. Gauss had an advantage over his colleagues in being a skilled calculator. That was an important asset for a mathematician in his time but is of little use today. No matter how good a calculator you are you can buy a better one for a few dollars.

Computer technology allows us to automate many of the simpler intellectual skills such as calculation. Inevitably this lowers the value of those skills while opening new possibilities to those with different skills. We are just beginning to understand what can be done and still view this opportunity too narrowly. We want to automate mathematical proofs so we try to create completely automated theorem provers. We want to automate chess so we try to make a computer program that can beat a grand master. Technology is far from being able to replace the human mind. The enormous calculating power of modern computers may soon be sufficient to defeat any human chess player with the brute force methods that such chess programs use. That is not the way to make the best chess player. To do that combine the special skills of the computer with the subtle skills of the human. Let the human use a computer program to aid play just as you let a student use a calculator during a physics exam. The best computer aided chess player will almost certainly not be the same person as the best unaided chess player.

Finding the worlds best computer aided chess player may not be important to cultural development but effectively using the computer to amplify human mental skills is. This is starting to happen with intuitive graphical user interfaces, programs to do mathematical analysis as well as computation and tools for scientific visualization. However we must recognize how primitive our understand-

ing is. People with powerful intuition that have played a major role in science like Einstein and Jung are usually in Jung's terminology thinking types. Their greatest strength is their powerful intuition but it is only through the dominance of intellect that they are able to digest the fruits of that intuition to a form that can be appreciated by our intellectually dominated culture. To get beyond this stage is no small task. We have *regressed* in the institutional structures to develop intuition since the middle ages. It is not possible for anyone to say what a world with intuition and intellect in more equal roles would be like other than it will be markedly different and far richer than the world we know. In some ways it will be more like the middle ages when there was not the extreme imbalance that exists today.

Chapter 7

The shadow of intellect

In Chapter 6 I said intuition is the key to developing the other dimensions of the psyche and the failure to deal with the shadow side of our intellectually skewed culture is a major contributor to the tragedies of this century. When we do not consciously integrate those elements of our nature they are still present. Instead of recognizing them in ourselves we project them onto others in an exaggerated form. Jung's work is the best source for understanding this but the bible puts it most succinctly.

And why beholdest thou the mote in thy brother's eye,
but considerest not the beam that is in thine own eye. (Mathew
7:2-3)[31]

Political skill and feeling

Everyone understands the need for people skills and recognizes that politics plays a role in all social activity. What we do not see is the fundamental conflict between thinking and feeling and how poorly developed and inferior feeling is in Western society. Feeling is a different and *equally* valid way of approaching and organizing experience. Jung defined wisdom as the union of thinking and feeling. They are complementary to each other and each is *essential*.

True feeling is the opposite of manipulative political skills. The following is from Jung's description of extraverted feeling.

I may feel moved, for instance, to say that something is “beautiful” or “good” not because I find it “beautiful” or “good” from my own subjective feeling about it, but because it is fitting and politic to call it so, since a contrary judgment would upset the general feeling situation. A feeling judgment of this kind is not by any means a pretense or a lie, it is simply an act of adjustment. A painting, for instance, is called beautiful because a painting hung in the drawing room and bearing a well-known signature is generally assumed to be beautiful, or because to call it “hideous” would presumably offend the family of its fortunate possessor, or because the visitor wants to create a pleasant feeling atmosphere, for which purpose everything must be felt as agreeable. These feelings are governed by an objective criterion. As such they are genuine and represent the feeling function as a whole(595)[25].

Something is beautiful because to see it as beautiful is the appropriate response to enhance what is most valuable. The beauty comes from the situation and not the painting. One does not see a “hideous” painting and lie about it for manipulative reasons. Rather what one says about what one sees is determined by a value judgment of the situation. In a different situation the same painting may be said to be hideous because that is the appropriate comment. If a response based on thinking were to call the same painting beautiful in one situations and hideous in the other it would be lying and hypocritical. This is not true of the feeling based response because the meaning of the perception is *defined* by the values in the situation. Of course someone oriented primarily be thinking interprets the feeling based response in terms of her or his own psychology and sees the actions as hypocritical.

Neither approach is correct or incorrect. They are both adaptations that work in some situations and fail in others. Nothing we perceive is absolutely objective.

Good feelings and a consistent intellectual framework are equally critical to the functioning of any institution or social organization. At the same time they are incessantly at odds. If thinking

dominates then feeling is often dealt with in manipulative and dishonest ways. This is legion in our culture today. Much of modern advertising is a perversion of feeling. If you want to be loved use our product. It will bring you happiness and fulfillment. Of course it never does and the more one pays attention to such nonsense the less happy and fulfilled one becomes. As we focus on an increasingly narrow intellectual model that reduces all values to a price tag, feeling revolts in the most grotesque ways. Gangs are one symptom of this. They are feeling based subcultures. Teenage pregnancy is another symptom. A mother's relationship to her child is the most intense feeling relationship. To have a child in circumstances when you cannot properly care for it is a perversion of that feeling, but one that can arise when other outlets for feeling are blocked, distorted or perverted. The most recent and grotesque example are the private militias. The bombing of the Oklahoma federal building is an act of evil but it is a mistake to see it as an isolated event. It is in part a product of the evil in our culture. We must find and punish those responsible, but unless we start to correct the evil in us that makes such a monstrosity possible there will be increasing violent and destructive reactions.

The arguments the militia make against the government are what Jung described as thinking governed by feeling.

... it does not follow its own logical principle but is subordinated to the principle of feeling. In such thinking the laws of logic are only ostensibly present; in reality they are suspended in favor of the aims of feeling(833)[25].

The members of the militia know something is horribly wrong. They do not and cannot understand it but they must lash out against it. Their ideas about how to respond originate in this feeling judgment and not in any objective information or rational understanding of the situation. To deal with the problem all of us must come to terms with the real problem. There is no intellectual model on which we can organize society. As long as we insist on our formulas for this, *no matter what the formulas are*, we will move closer to disaster. Neither Capitalism, Socialism, Communism nor any other 'ism' in the sense of a conceptual system will work for long. Society

must be rooted in human and religious values. That is the inspiration for the declaration of independence and the constitution and it is an inspiration that is badly in need of renewal. The militia do not miss the mark when they speak of our straying from these documents as the source of the problem.

It is ironic that feeling at its most grotesque and perverted can bring out feeling at its best. For it is in times of tragedy that we most often see feeling at its best in our culture. Attitudes become, for the moment, dominated by people's concern for each other. Personal agendas take a back seat. People soothe and comfort each other because that is what is most important and not for any ulterior motive.

It is the spirit of feeling at its best that we must renew through a commitment to human and religious values. We must continue to use all that we understand about economics and motivation but we must do this in a context of values that cannot have its roots in economics or in any conceptual system.

Intuition as a path to integrating feeling

In an individual or a culture it is most difficult to develop the function opposite to the dominant one. Thinking and feeling are both rational functions but rationality can only have one orientation at a time. If it is oriented towards a conceptual framework it cannot be oriented towards valuing. There is not the same opposition between intuition and thinking. They are complementary ways of dealing with the same material. Intuition has played an essential, if somewhat subterranean, role in the achievements of intellect. The path to feeling must be through an expansion of intuition. When intuition plays a dominant role it can arbitrate between feeling and intellect. As we develop and differentiate intuition we can begin to allow it to dominate at times and in so doing we provide an opportunity to develop and differentiate the shadow side of thinking.

To the degree that the shadow remains unconscious and undifferentiated we project it onto others in ways that distort reality. Rather than deal with the inferior and sometimes malevolent aspects of ourselves and our nation we blame the victims of that

malevolence. If those that succeed do so on their skills and competence and those that fail were given a fair chance and could not cut the mustard, then it is too bad for them. The solution is to build more prisons and increase the penalties for all types of crime. The solution is to deport illegal immigrants. The solution is to stop subsidizing the welfare bums. At the same time that we do these things we change the economic system so most of the poor and an increasing percentage of the middle class have little or no chance for a decent life. This is a perfect recipe for creating hell on earth.

These policies are the product of the undercurrent of inferior feeling in a thinking dominated society. The mere fact of one's success is evidence of one's superiority. The people stabbed in the back along the way to success were losers any way. The more wealth and power one acquires the more one wants and the more this is evidence for the justice of one's position. It is quite impossible to objectively evaluate what one has contributed and what one has taken.

How do we get out of the hole we are furiously digging for ourselves? It will not be easy. I fear that it is going to take greater tragedies than we have already experienced. It is impossible for a nation to control its people, but intellect likes to control things. A nation must nourish its people or the people will devour the nation. If instincts for control prevail the tragedy we are creating will be enormous.

The solution is an expansion of consciousness. We must develop and use intuition as a coequal of intellect. We need a basis for deciding when human values must take precedence over the tenets of an incomplete intellectual model. We need a referee between intellect and feeling and that referee at this stage can only be intuition. Intuition can help us recognize the long term consequences of an action. It can go beyond the tenets of *any* intellectual system to recognize what consequences and values are beyond the reach of the system.

Intuition is not itself the solution but it is the opening to the other dimensions of the human psyche that are the key to the solution. Inferior feeling is the most obvious defect we have but it is by no means the only one. Intellect will not be supportive. It knows what it knows and it does not trust things that are impre-

cise or beyond its capacity to use. We must make an end run around intellect by showing how limited an intellectual approach is. I hope my approach to physics and mathematics will become a part of this process. The alternative is for culture to push an intellectual approach to the point that it creates enormous tragedies. This would lead to intellect becoming the rejected shadow. Such regression would be extremely dangerous in a world with modern technology. Militias, gangs extreme fundamentalist religions and cults are all examples of where that process leads.

Chapter 8

Human institutions

There is a dimension to the human spirit that can never be satisfied. It is not something that has a definable goal or end point such as the craving for food or sex. It is always striving for something new even if what one has seems completely satisfactory. Why would evolution develop an instinct for creativity? By investing some resources in exploring new possibilities *just for the sake of exploring* one occasionally hits on something that is of practical practical value. The roots of this instinct no doubt go back to how mutations are dealt with at the most basic level. More often than not mutations are bad but the best strategy is not to automatically eliminate them. Rather it is to allow a few to remain and see what happens.

Sexual reproduction was a great advance because it provides a means for trying out an enormous number of combinations of genes. Sex is a method to both speed up creativity and to support a higher level of diversity in a population. The latter can be important to adapt to recurring situations and the former to adapt to new situations. Just as introversion and extraversion have their roots in two basic strategies of survival the instinct for creativity and the freedom that is necessary for creativity have their roots in the creative mechanisms that are a part of reproduction.

There are potentially great advantages to an organism if it can build a model of part of its environment and, even to a very limited degree, predict how the environment will change over time. Higher animals invest many resources in sensation and in a nervous system to draw inferences from sensation. Gödel's proof applies to

this process. Higher animals are capable of higher levels of abstraction that determine a level of mathematical processing that the organism is capable of. At a low level the nervous system develops proportionate responses. If a prey is moving fast one strikes fast and in a direction and with a speed determined by the motion of the prey. At a higher level there is pursuit. One may not be able to capture the prey in a single swift blow but one follows it waiting for an opportunity. Pursuit provides the opportunity for more subtle strategies such as subterfuge and tiring the prey. Most importantly it opens the door to cooperation. Gödel's result suggests that only a divergent process like evolution would be likely to evolve the mathematical sophistication of the human mind. Survival, by itself, is not enough. If the planet was smaller with less diverse climate then perhaps the development of anything approaching the human mind would be unlikely. Perhaps periodic global catastrophes like the event that destroyed the dinosaurs also played an essential role. By destroying less flexible life forms these catastrophes opened the way to greater diversity in the more flexible and creative species that remained.

In Chapter 3 I described how the mathematics of the completed infinite might be interpreted as the mathematics of creativity. Is it possible to apply this mathematics to quantify constraints on evolution? I believe this is possible and will happen in time. Can we apply the same mathematics to human institutions in which creativity plays such a vital role? I think this will happen also. We are a long way from being able to do any of this in a quantitative way. However we can begin to relate some of the things we already understand about creative institutions to the relevant mathematics. This may provide some insights of practical value now as well as point the way to developing a practical mathematics of creativity. In time we may be able to answer some political questions with mathematical or scientific rigor. They have calculable answers or answers that can be determined through experiments.

The ideas that can lead to such achievements are in a primitive state. We can begin to understand the tradeoffs between diversity and complexity in nature and how we might be able to quantify them not simply in terms of optimizing survival under fixed conditions but under conditions that change over time in an unknown

way. That is the problem that evolution is faced with and has solved. In time these ideas can be made more rigorous and techniques can be developed to quantify some of these relationships. The mathematics of creativity is a profoundly important practical science because creativity is the principle business of life.

Stability and creativity

May you live in interesting times. — Chinese curse

There is increasing evidence that evolution proceeds by what is called punctuated equilibrium. For long stretches a species remains essentially unchanged and then, during a brief period, there is rapid development to a new species. The conditions for this may be the isolation of a part of the population or some new factor that affects the entire population.

Change is painful, disruptive and dangerous. Stability is pleasant and nourishing. We have suffered the curse of living through the most rapid and dramatic changes in history. It is extremely difficult to adapt to such rapid change. We have no choice. We crave stability but we cannot reach stability until we go through the disequilibrium of this time. The stability that we move towards must be more broadly based than our consciousness is today. Broadening consciousness in an individual is heroic difficult work. Doing the same in an entire nation or planet is heroic and difficult on a global scale. The work proceeds one individual at a time. The consciousness of a nation is invested in the consciousness of its people. Broadening an individual consciousness as part of a global broadening of consciousness is what nature and the times demand of us. If we are not up to the task the danger that we will self-destruct as a species is quite real. Evolution will try again if it has to.

Intellect is leery of broadening consciousness for this requires methods and ideas that are beyond intellect's grasp. There is no prescription or formula for doing this. We must pay more attention to our inner nature. We must spend time in quiet thought or a walk through the woods. We must read and let what we read soak

in rather than just being understood. We need serious dialogue with others and with ourselves.

At the time when this is most needed external forces conspire to make it most difficult. Our jobs demand more time and concentration. The media bombards us with empty entertainment and shallow ideas. What little leisure time we have is devoted to carefully structured exercise or other organized activity. We are in a self destructive feedback loop. The harder we try the worse things get because trying harder is a big part of the problem.

We need to recognize the poisonous nature of these forces and we need to find ways to circumvent them. Money and power are at the root of these forces. Wealth and power are essential. Without them we can do nothing. However the struggle for power is becoming trivialized as a struggle for financial success. Money is the measure of all things. It is no longer just a tool. Politician talk about investment as if all investment involved financing corporate activity. It is the ultimate absurdity that we cannot see the economic value in investing in the education, development and security of our children while we spend a fortune escalating without end on new prisons.

This is intellect pushing past the point that it works. Money is a simple quantitative measure of success. It is a goal intellect can focus on, but it is an empty goal that leads nowhere. Without a religious sense of connectedness to something beyond one's individual existence nothing has an enduring meaning. Intellect has smashed all the fairy tales of religion and is too narrow and blind to see past the fairy tales to the underlying truths. It has nothing to guide it but a calculation of net worth.

A revival of religious values is essential. This cannot be a regression to the old fairy tales. We must renew the eternal values with a vision of religious truth that is compatible with the achievements of modern science. That is perhaps the greatest value that can come from Jung's work.

Wealth is the collective product of a nation and mankind. Our wealth in large measure is the legacy of knowledge and culture from those who came before us. Whatever an individual or generations adds to that is minute. That legacy belongs to every human being equally just as the right to breathe the air does. Of course

there is a role for private property, but that role is to provide the resources and motivation to contribute to our collective wealth of culture and material goods. Property rights are *always* subordinate to human rights. When private wealth goes past the point of providing needed resources or motivation for productive and creative activity it has no purpose. To see wealth itself as a measure of success or achievement is corrupt and destructive.

We should see the distribution of wealth as an optimization problem. How do we structure an economy to maximize wealth in the broadest sense of that term? We should not look at it in terms of who is entitled to what. Wealth would not exist without society and nation and the distribution of wealth in those institutions has always been highly arbitrary. Of course one must respect the current state of affairs. What ever changes are made must evolve from the state we are at in a way that is not too disruptive.

Optimizing productivity and creativity

In this section I suggest a few practical ideas to help us break out of the destructive feedback process we are in. These ideas are not original. They are compatible with and to a small degree justifiable through the mathematics of creativity.

Creativity requires resources. Invest too many resource in creative activity and you fail to meet basic needs. Soon resources disappears and creative activity as well. Invest too little resources and your rivals will soon be so far beyond you that you become irrelevant.

We need large companies and large projects to achieve certain goals. At the same time we recognize that creativity and innovation seems to thrive best in smaller companies and projects. If total wealth is increasing then it should both be spread more widely and provide more opportunity for individual institutions to become larger. We need to get some quantitative measure on this tradeoff. I strongly suspect that we are currently skewed towards excessive concentration of resources.

Taxes and incentives should be structured so most can build up enough personal wealth to have some stake in society and ul-

timately some degree of financial independence. A nation of wage slaves is better than a nation of slaves but far worse than a nation of independent people. Immense personal fortunes serve little purpose. There ought to be a tax on wealth (not income but wealth) past a certain limit based on per capita net worth. For example it might take effect when an individual's net worth exceeds some multiple of per capita net worth. This tax should be progressive. If that destroys the incentive to produce more wealth in a few individuals that is likely to be all to the good. When personal wealth gets past the point of providing needed resources and financial security it is likely to do more harm than good.

Large companies and projects limit diversity and thus creativity. Of course they have other compensating advantages. There should be a bigness tax beyond a certain point. This tax could always be avoided by splitting into truly independent companies. We could experiment with the limit to see how it should be set relative to national net worth.

For most of the life of our nation democracy provided a constraint on the excess of large institutions. Every person has exactly one vote regardless of their wealth. The size of our country and the cost of media to contact large number of voters has significantly eroded this constraint. We need to restore this balance through campaign finance reform and perhaps other mechanisms.

Chapter 9

An infinite future

One sided development of intellect has created a poverty of values. This has led to a destructive feedback loop in our culture where more effort is required for intellectual motivated activity at the same time we most desperately need the space to broaden our consciousness. Worse, our enormous intellectual achievements are creating new challenges to our values and wisdom far beyond those of previous generations.

Religion and values

We are on the verge of being able to consciously control human genetics. One can foresee a day when we can create a baby to specifications. The first reaction of traditional religion is not to do it. Thus the Catholic church with the logical consistency that is characteristic of that institution draws the line at *in vitro* (in glass or more generally outside the body) fertilization. That is a consistent way to deal with the dangers presented but it is one that will ultimately fail. Too much can be accomplished that is obviously beneficial. The development of these capabilities cannot be stopped. It is essential that we develop religious values that can deal with the enormous new power these techniques present.

This is a necessary and inevitable step in evolution, albeit an enormously dangerous one. Just as evolution over the eons teaches us how to consciously use more of the resources of earth she in-

evitably will teach us to use and control evolution. Religious leaders are correct in urging caution. We are on the verge of taking conscious control of processes that are governed by unconscious laws developed over the eons. Without consciously acquiring some measure of the wisdom innate in those laws we will make a terrible mess of it. Wisdom is not in abundance in our institutions. A small part of that wisdom can come from the mathematics of creativity. This can teach us a few of the fundamental constraints that evolution operates under and provide specific guides on what is definitely a mistake and what is necessary to preserve creativity.

Religious values do not come from mathematics. Our values will determine how we use our mathematical understanding and if this new power will be our undoing as a species. We can not create or control our values but we can develop them and sensitize ourselves to them.

Evolving structure

We have only the slightest idea of the boundless mathematical abstractions that may evolve in time and lead to ever richer mathematical systems. We are incapable of imagining the *experience* that will evolve as we use this knowledge to extend the scope of evolution. At any point in time we can only know an infinitesimal fraction of mathematical truth. At any point in time the experience that we have is the merest hint of a shadow of what can be. God is the never ending realization of that potential.

Time in a finite model or one that has completed infinite structures is the same as space. There is nothing special about the time dimension. This is not necessarily true of a potentially infinite universe. Such a universe may start with nearly uniform and simple conditions over some region. Over time it can evolve structures of arbitrary complexity spanning arbitrarily large regions.

Creativity is tied to an arrow of time. Creativity produces the structures that are the experience of time. Time is how the finite approaches infinity while always being infinitely far from it. Of course we can never know but the most interesting fantasy assumes the creative process is unlimited.

Who am I

A baby learns to differentiate I from not I as a practical necessity. I is what can be controlled by sheer act of will. Not I, such as parents, require more subtle or devious means. The distinction comes from the wiring of our nervous system and the mechanics of the body. There are no absolute or metaphysical boundaries between I and not I. The I of the nervous system was conceived, born and will die. 'I' in this sense is ephemeral.

We choose what we mean by I. Love broadens our sense of self. Love can be personal, or religious. Our sense of self can extend to our families and to life itself. 'I' in this sense is eternal.

Experience is particular and at the same time whole. Our individual lives are particular and at the same time an inseparable part of a wider unity. We experience this wider unity as 'I' whenever we feel our connectedness to and unity with others and with life.

I walk through the towering redwoods in a three dimensional landscape of sharply cut gullies falling towards a rushing stream. How many years have I walked this trail? Fifteen? As long as the land has been here? Am I here? No, here is I.

Appendixes

Appendix A

Set theory

In mathematics a formal system is a set of axioms and the rules of logic for deriving theorems from those axioms. It can be thought of as a computer program for outputting theorems. We can write a program to output all the theorems for *any* formal system. The axioms say what primitive objects and relationships exist and how new objects can be constructed.

In set theory there is one primitive object, the empty set, and one relationship, set membership. All of mathematics can be modeled with these primitives. For example the integer 1 is defined as the set that contains the empty set. The integer two is the set containing 1 and the empty set. Integer N is the set containing the empty set and all integers less than N .

The most powerful generally accepted formal system is Zermelo Fraenkel (ZF) set theory. We will list the axioms of ZF adapted from Cohen(50)[11]. First we need to explain the notation. Sometimes we refer to arbitrary statements in the language of ZF with upper case letters. A refers to any valid statement in the language of ZF. $A(x)$ is a statement with a single parameter. $A(t_1, \dots, t_k)$ is a statement with k parameters.

A.1 Notation

- ⊃ This is the primitive membership relationship. $a \ni b$ means a is a member of b .

- \nexists $a \nexists b$ means a is *not* a member of b .
- \forall This is the universal quantifier. $\forall x A(x)$ is true if and only if $A(x)$ holds for every set x .
- \exists This is the existential quantifier. $\exists x A(x)$ is true if and only if there exists at least one x such that $A(x)$ is true.
- \forall_x This is the universal quantifier restricting x to all elements of the set z .
- \exists_x This is the existential quantifier restricting x to search for elements in the set z .
- $\exists!$ $\exists! x A(x)$ is true if and only if there is one and only one set x such that $A(x)$ is true.
- \equiv This is logical equivalence. For any two statements A and B $A \equiv B$ is true if and only if A and B are either both true or both false.
- \rightarrow This is logical implication. For any two statements A and B $A \rightarrow B$ is true if and only if A is false or B is true. In other words if A is true then B must be true.
- A_n We can write a program to list all the statements in the language of ZF. A_n refers to the n th statement output by such a program.
- \bar{A} This is the negation of A .
- \subset $A \subset B$ means every member of A is a member of B and B contains at least one member not in A .
- \subseteq $A \subseteq B$ means every member of A is a member of B . A and B may be the same set.
- $\{x\}$ If x is a set then the set containing x is written as $\{x\}$.
- \cup If x and y are sets then the union of x and y (the set containing those sets and only those sets in x or y) is $x \cup y$.

$0, 1, 2, \dots$ In ZF the empty set is the integer 0. The integer n is the union of the empty set and all integers less than n .

ω ω is the union of all integers.

A.2 The axioms of ZF

1. *Axiom of extensibility*

$$\forall x \forall y [(\forall z (z \ni x \equiv z \ni y)) \rightarrow x = y]$$

Sets are uniquely determined by their members. There cannot be two different sets that have the same members.

2. *Axiom of the empty set*

$$\exists x \forall y [y \not\ni x]$$

There is an empty set that contains no set.

3. *Axiom of unordered pairs*

$$\forall x \forall y \exists z (\forall w) [w \ni z \equiv (w = x \vee w = y)]$$

For any two sets there is a third set that contains those two sets and only those two sets.

4. *Axiom of union*

$$\forall x \exists y \forall z [z \ni y \equiv (\exists t (z \ni t \wedge t \ni x))]$$

For every set x there is a set y that contains the union of the sets that are members of x . In other words for every set x there is a set y such that t is a member of y if and only if there is some z that is a member of x and t is a member of z .

5. *Axiom of infinity*

$$\exists x [0 \ni x \wedge \forall y (y \ni x \rightarrow y \cup \{y\} \ni x)]$$

There is a set ω that contains the empty set and if any set y is in ω then the set containing the union of y and the set containing y is also in ω . By induction ω contains every finite integer.

6. *Axiom of replacement*

$$\forall x [\exists ! y A_n(x, y)] \rightarrow \forall u \exists v (B(u, v))$$

where $B(u, v) \equiv [\forall r(r \ni v \equiv \exists s[s \ni u \wedge A_n(a, r)])]$

This is an infinite set of axioms, one for each statement $A_n(x, y)$ in the language of ZF. We must expand $A_n(x, y)$ to $A_n(x, y, t_1, \dots, t_k)$ and surround the statement with universal quantifiers on ' t_1, \dots, t_k '. We can enumerate all the $A_n(x, y, t_1, \dots, t_k)$ but we cannot enumerate all the sets t_i that can be specified as fixed parameters in a statement. Thus we cannot index all statements with all parameters with the integers. We must use quantifiers ranging over all sets to include every possible statement in the language of ZF with every possible value for a fixed parameter.

The axiom of replacement allows us to make a new set v from any statement in the language of ZF (with any fixed parameters) that defines y uniquely as a function of x and any set u . A set r belongs to v if and only if there is a set s that belongs to u and r is the image of s under the function defined by $A_n(x, y)$.

7. *Axiom of the power set*

$$\forall x \exists y \forall z [z \ni y \equiv z \subseteq x]$$

For any set x there is a set y that includes every subset of x . This is the axiom that defines sets of higher cardinality or at least seems to. All the subsets of a set definable in a given formal system does not include all possible subsets. In fact the sets definable in a formal system are recursively enumerable and thus countable no matter how pretentious the axioms of the system are.

The important thing to understand about the axioms is that they are comparatively simple precise rules for deducing new statements from existing ones. One can easily write a computer program that will implement these rules and print out all the statements that one can prove from these axioms. Of course some mathematicians think there is a Platonic heaven of all integers, all subsets of the integers and all true sets. They see these axioms as more than a formal system. They see the axioms as telling us about this idealized mathematical truth.

A.3 Ordinals and cardinals

Ordinals and cardinals form the backbone of mathematics. Ordinal numbers describe orderings. Cardinal numbers are the measure of size in mathematics.

They first ordinals are the integers. ω , the set containing all integers, is the ordinal of the integers. The next ordinal is the successor of ω . This set has one members ω just as the successor of 0 has one set, the set containing the empty set. The successor of $\omega + 1$ is the set containing ω and the successor of ω .

Ordinals can be thought of as general way of representing induction or iteration. ω corresponds to iteration that can be characterized by a loop up to any integer or induction on all the integers. $\omega + \omega$ contains ω and all finite successors to ω . $\omega \times n$ contains all finite successors to $\omega \times n - 1$. $\omega \times \omega$ contains $\omega \times n$ for all integers n . $\omega \times \omega$ corresponds to two nest loops each with a limit of any integer. ω^n corresponds to loops on the integers nested n deep. ω^ω corresponds to loops on the integers nested n deep where n is a parameter. All forms of iterations and all induction can be characterized by some ordinal number. By using infinite sets to describe iteration one masks over the rich combinatorial structures that are required to define higher levels of iteration. Recursive iteration is characterized by the recursive ordinals but there is no recursive algorithm to describe the structure of all recursive ordinals although there is such an algorithm for any recursive ordinal.

Each integer is a finite cardinal number. Different integers have different sizes. ω is the cardinal number of all integers. No one knows what the next largest cardinal is. Cantor proved that the Cardinal of the reals is larger than the cardinal of the integers by showing that there could not exist a one to one map between the integers and reals. Gödel constructed a model for set theory in which the first cardinal larger than the integers is the cardinal of the real numbers. Gödel proved this model was consistent of ZF set theory was consistent[19, 20]. Cohen proved that if ZF set theory is consistent that it is consistent to assume there exists cardinals greater than the cardinal of the integers but less than the cardinal of the reals[11]. This question is known as the Continuum Hypothesis and is undecidable in ZF.

If there do not exist completed infinite totalities than there does not exist a single real number let alone the set of all numbers. In that case the set of all real numbers only make sense relative to a given axiom system. The continuum hypothesis is true, false or unprovable in a given formal system, but it has no absolute truth value.

A.4 Recursive functions

In mathematics we express operations that we cannot possibly perform. Consider the statement $\forall x \exists y A(x, y)$. To determine if this statement is true we would need to take every integer x and see if there exists any integer y such that $A(x, y)$ is true. If there is any integer x for which we cannot find such a y the statement does not hold. Clearly there is no general way to determine if such a statement is true. We can define functions in ZF through relationships of the form $A(x, y)$ where A defines a unique y for every x . However if A contains any quantifiers then there is no general way to compute the function. In the 1930's several proposals were made to characterize what is meant by a mathematical algorithm, i.e., a mathematical procedure that could be carried out in a finite number of steps to compute any value of a function. These attempts were all shown to be equivalent. The fascinating thing about this is that we need so little machinery to fully characterize mathematical algorithm. Any computer, if it had access to potentially infinite storage, would qualify as a universal TM. That is for any algorithmically computable or *recursive* function there is a program for that computer that will compute each value of the function in a finite time.

An important implication of this is that we can *Gödel number*, i.e., assign a unique integer to every possible recursive algorithm. All we need to do is concatenate the bits that describe the program Programs typically have parameters and we can arbitrarily divide up the memory into program area and data area. To allow for arbitrarily large programs and arbitrarily large input we can think of the memory as an infinite tape with a center mark. The program is written on one side of the mark and the initial parameter on

the other. For any specific program and parameter we can easily compute the Gödel number of a program with no parameter that will perform exactly the same steps. The distinction between input and program is arbitrary. We can just as easily encode the input in the program. We define R_n as a numbering of all programs with no parameter and P_n as a Gödel numbering of programs with a single parameter.

Theorem. There can exist no program (recursive algorithm) D with a single parameter n such that $D(n) = 1$ if R_n halts in a finite time and $D(n) = 0$ if it never halts. This is often referred to as the halting problem.

Assume there is such an algorithm. We can use this algorithm to construct a new function $D_1(n)$ such that $D_1(n) = 1$ if $P_n(n)$ halts and $D_1(n) = 0$ if $P_n(n)$ loops forever. We have a specialized version of the halting problem for programs that have their own own Gödel number as input. If we can solve the general halting problem we can solve this specialized version. The specialized version allows us to apply a simple version of the diagonal argument that is crucial to Gödel's proof and many related proofs.

From $D_1(n)$ we construct $D_2(n)$ that loops for ever if $D_1(n) = 1$. We can compute m such that $D_2 = P_m$. Consider $D_1(m)$. By the definition of D_1 , $D_1(m) = 1$ if and only if $P_m(m)$ halts. From the construction of D_2 , $D_2(m)$ (which is $P_m(m)$) loops forever if $D_1(m) = 1$. Thus we conclude that $P_m(m)$ loops forever if $P_m(m)$ halts. The assumption that $D_1(n)$ exists is false and the theorem is proved.

Definition 1 A set of integers S is recursive if and only if there is a recursive function $S_d(n)$ that yields a value of 1 if $n \ni s$ and 0 otherwise. Note S_d must halt in a finite time for every input n .

Definition 2 A set of integers S is recursively enumerable if there is some recursive function $S_e(n)$ such that $m \notin S$ if and only if there exists some k such that $S_e(k) = m$. A set is recursively enumerable if it is the range of a recursive function. Each element of the set can be enumerated in a finite time by successively executing S_e on each integer input.

Theorem. There is a set that is recursively enumerable but not recursive.

The proof is trivial once one understands that one can with a single recursive process simulate all recursive processes. For example one can for each n simulate the first n recursive processes for n steps. Eventually you will simulate every step for every recursive process. As soon as one of these halts you can output the Gödel number of this process. Thus the Gödel numbers of recursive processes that halt are recursively enumerable. We have already proven that this set is not recursive. We cannot decide with a recursive algorithm what numbers do *not* belong to this set.

A.5 Incompleteness theorem

This discussion of Gödel's proof does not follow Gödel's constructions or formulation. It is extremely informal and uses understanding of computer programs to make the ideas that underly Gödel's results more easily accessible to a contemporary audience.

Recursive functions are good because we can, at least in theory, compute them for any parameter in a finite number of steps. As a practical matter being recursive may be less significant. It is easy to come up with algorithms that are computable only in a theoretical sense. The number of steps to compute them in practice makes such computations impossible.

Just as recursive functions are good things decidable formal systems are good things. In such a system one can decide the truth value of any statement in a finite number of mechanical steps. Hilbert first proposed that a decidable system for all mathematics be developed. and that the system be proven to be consistent by what Hilbert described as 'finitary' methods.[16]. In response to this challenge Gödel developed his famous theorems known as the first and second incompleteness theorems. These show no such formalization is possible for non trivial consistent systems.[16]. He went on to show that it is impossible for such systems to decide their own consistency unless they are inconsistent. Note an inconsistent system can decide every proposition because every statement and its negation is deducible. When I talk about a proposition being decidable I always mean decidable in a consistent system.

One key to Gödel's proof is the 'Gödel numbering' of all the state-

ments in a formal system. This is an algorithm to assign a unique integer to every statement in the language. Today this is simple and intuitive. Today we convert computer programs and just about everything else to Gödel numbers as we store them as bit patterns in computers. In Gödel's time it was a stroke of genius. Gödel numbering things is often thought of as an abstract mathematical idea, but it is both extremely common and immensely practical.

Once we have Gödel numbered the statements in a formal system we can think of the system as a recursive process for enumerating the numbers that correspond to provable statements in the system. If a system is strong enough to define any recursive function it must be strong enough to define itself as a recursive process. That is within the formal system we can define a recursive process R_n that enumerates the Gödel numbers of all the statements that can be proven in the system. Kleene points out that Gödel's proof constructs within the formal system S he is working with a statement that says "I am unprovable in S "(128)[16]. Of course if this statement is provable in S then S is inconsistent. So any system strong enough to model itself in this way and to construct this statement must be either incomplete or inconsistent. It turns out that this is almost any nontrivial formal system.

How do we construct this self referencing statement. Gödel's proof does this in detail and is complex for this reason. We can understand informally how to do this construction by understanding that a formal system, S , can model itself as a computer program, R_s , that outputs the Gödel numbers of theorems. (Today we think of programs as outputting text like the text of theorems but this is still Gödel numbering. The text is output as ASCII character codes or another bit pattern coding for characters.) The statement that a particular theorem is provable is the statement that R_s will output a particular number. The Gödel number for the statement R_s does not output x will not ordinarily be x . However, by using a sort of diagonalization technique, we can construct a statement that says what we want.

Let us define $A(x)$ to be the statement that if x is the Gödel number of a statement $S_x(y)$ in system S with one free integer variable then $S_x(x)$ is true. We are treating x as both the Gödel number of a statement ($S_x(y)$) and an integer. We apply the statement to this

integer by setting y in $S_x(y)$ to x . This is straightforward although a formal proof requires construction of the Gödel numberings and other details. Assume a is the Gödel number of $A(x)$. $A(a)$ is the statement we want. It asserts that it cannot be derived from S . If we could derive it from S it would be false and thus we would have a contradiction. Thus $A(a)$ is true. Note we have just derived $A(a)$ from the assumption that S is consistent. This implies that we cannot prove the consistency of S within S itself. If we could we could derive $A(a)$ and this would imply that $A(a)$ is false. This result is Gödel's second incompleteness theorem.

Gödel's proof is another in the history of proofs based on self reference going back to the ancient liar's paradox. The essential new ideas are Gödel numbering of statements and modeling a formal system within itself as a computer program to enumerate theorems (or Gödel numbers of theorems).

Another way to prove Gödel's first incompleteness theorem is to use the result that we cannot in general decide for a recursive function F if some number x is in the range of F , i.e. there is a set that is recursively enumerable but not recursive. We proved the existence of such a set in the previous section and gave the example of the halting problem for computer programs. A formal system that is strong enough to state the halting problem for all programs cannot be decidable. There are statements of the form program x never halts that are true but not decidable in the system. If all such statements were formally decidable then we could decide the halting problem by enumerating all theorems. Eventually (in a finite time) one of the theorems would be that the program does halt or that it does not halt.

Appendix B

Physics

This appendix gives a minimal introduction to physics to aid in understanding Chapter 4 and Appendix D. No attempt is made to provide a complete or comprehensive development. The purpose is to make this book relatively self contained for someone with a high school level understanding of physics and algebra. It is hoped that this will be a spur for further reading for anyone not familiar with these topics.

B.1 The birth of new physics

In the early part of this century classical physics seemed to be nearly complete. It provided a elegant intuitively satisfying model that could be used to solve a vast array of physical problems with precision. There were a few loose ends. The Michelson Morley experiment failed to detect the ether expected to be the medium of propagation for light. The black body radiation anomaly remained unexplained. In a short time these loose ends would lead to the two great revolutions in twentieth century physics of relativity and quantum mechanics. The birth of quantum mechanics in contrast to relativity involved a long labor and a great deal of pain. The explanation of the black body radiation anomaly lay in the quantization of radiation. Light could not be emitted in arbitrary amounts. The energy was quantized so that the minimum radiation at a given frequency ν was $h\nu$ where h is Planck's constant.

B.2 Quantization

This was very strange. In classical physics radiation is the oscillation of a medium of propagation. Sound is the oscillation of air. There is no problem in creating a wave of arbitrary amplitude at any frequency. This is not true of electromagnetic radiation. It comes in minimum size chunks as if it were a particle. Increasingly it became clear that whatever exists at the quantum level it is unlike any existing physical objects or mathematical models. The fundamental building blocks of nature all have a frequency like classical waves and all come in discrete energy packets like particles.

Things got much worse. A classical wave cannot have both a definite position and a definite frequency. Only an impulse has an exact position and an impulse is the integral over all frequencies. But classical waves are not quantized. The wave nature of the fundamental particles prevents us from simultaneously determining the position and momentum of a particle. If we prepare the wave so it has an accurate position we will spread out the momentum and vice versa. How is one to predict what happens with these particles that have a frequency? There seems to be no way to predict exactly what will happen. There is an equation that describes the exact evolution of the quantum mechanical wave function. However we cannot use the wave function to predict what is observed. All we can get from it is the probability that we will get a particular observation. Worse once an observation is made we must use that observation to constrain the future evolution of the wave function. Worse we can only model the single particle wave function in physical space. If we have an N particle system then we must model the evolution of the wave function in a space in which there are a separate set of spatial dimensions for every particle. For N particles there are $3N+1$ dimensions. This is called configuration space. Worse once two particles interact they become entangled. An observation of one particle can constrain observations of another in a way that can only be modeled if information about that observation propagates nonlocally so it can influence the second observation.

In other words the theory is an absolute mess. Einstein felt the more successful the theory became the sillier it became. He

felt that we would need to start over with something completely different.

B.3 Bell's inequality

A photon is the minimum quantity of light that can exist at a given frequency. Light as a wave has a polarization or an angle at which the wave oscillates. A polarizer is a material that will let light pass completely if the polarization of the light is aligned with the polarizer and will block it completely if they are orthogonal. At other angles (θ) it lets some fraction ($\cos^2(\theta)$) of the energy of the light through. Polarizing sun glasses are effective in part because sunlight that is reflected off the surface of an object at a sharp angle is highly polarized. In classical physics the quantity of radiation that passes a polarizer is determined by the angle. If we have a single photon this will not work. Either the photon gets completely through or it does not get through at all. There is no way the photon can split into two parts one of which makes it and another that does not. Thus in quantum mechanics the angle of the polarizer determines the *probability* that the photon will traverse the polarizer. Further if the photon does traverse the polarizer its polarization will be exactly aligned with the polarizer.

Now consider a particle that decays into two photons. Conservation laws require that the polarizations of the two particles be exactly correlated. Quantum mechanics requires that the particles do not have a polarization value until they are observed.

This seems very strange. Something that cannot exist in either particle is exactly aligned between them. It gets stranger. Before either particle is observed we have no idea what the polarization is. If one particle traverses a polarizer and we then detect it the probability that the second particle will traverse its polarizer can be computed by assuming that both particles polarization are exactly correlated with the polarizer that the *first* particle traversed. For some combinations of polarizer angles, this correlation is so high that such results cannot be modeled unless the angle of the first polarizer affects the probability that the second particle will be detected. Experiments to test this such as that illustrated in Fig-

ure D.1 on page 126 have been performed. The time between when the polarizers change and when this has an effect is determined by the distance from a polarizer to the *closest* detector. In theory such an experiment could be spread over a billion light years and a polarizer setting a billions years away could affect a detection in an arbitrarily short time. That is what quantum mechanics predicts. Bell[3] and Eberhard[12, 13] proved this. Bell showed that correlated results that are space-like separated most obey a mathematical relationship known as Bell's inequality. Quantum mechanics predicts this inequality is violated. Two events are space-like separated if they are far enough apart in distance and close enough together in time that light cannot travel between them.

Does nature act this way? We do not know because none of the experiments to date are conclusive. Most physicists believe these predictions of quantum mechanics in part because they have a very strange property associated with the claim that probabilities are irreducible. One of the polarizers must influence one of the detections but we cannot tell which one. As a result the predictions are the same in any relativistic frame of reference. However no mathematical model can reproduce these predictions without operating in a preferred frame of reference in a way that violates relativity. Many physicists believe that something special is happening here that cannot be modeled by classical mathematics and that does not involve nonlocal effects. Perhaps they are right and perhaps they are rationalizing.

B.4 Relativity

In mathematics one makes the distinction between a manifold and a metric. A manifold is a topological structure of points. A metric is a distance function that describes how far apart any two points are. One naturally thinks of a metric as being solely determined by the manifold. That is one thinks that two points have a fixed distance between them no matter how the universe changes in other ways.

Special relativity shows that this is not true. The distance we measure between two points is affected by our motion relative to those points. Special relativity suggests, but does not require, that

there is no manifold of points or absolute frame of reference. There are only objects and relationships between those objects. This seems strange and unintuitive but it is certainly possible in a continuous universe. In a fully discrete universe it is not possible to do away with the underlying manifold and special relativity can only be approximately true in such a universe.

One way special relativity can come about is if all physical effects are electromagnetic. This is impossible in the existing theory because electromagnetic waves travel at the speed of light. If the theory had a small element of the correct sort of nonlinearity one could create stable soliton wave structures. Time and distance within such a structure is affected by the motion of the structure. Both the linear motion of the system and any internal dynamics are described by the same partial differential equation. For example if the structure were moving at the velocity of light it could have no internal dynamics. The only dynamics would be the motion of the object in space. In effect time would stop on the object.

The faster an object moves the slower its internal dynamics relative to an object at rest.

General relativity extends so that the distance we measure is also affected by the gravitational field.

Appendix C

Recursive structures for mathematics

[Note to editor. The technical work in this appendix is under development. It is possible this will lead to new results publishable in a mathematics journal. The book does not depend on such results and I can complete an abbreviated version of this appendix in a few days if needed.]

This appendix develops a recursive functional hierarchy as an alternative approach for the foundations of mathematics. Instead of speaking of arbitrary sets we will speak of the range of a recursive functional. This does not limit us to recursive sets because the domain of the recursive functional need not be a recursive set. For example the domain might be notations for recursive ordinals.

It may seem to require conventional set theory to define that domain. However one does not need to define the set of all notations for recursive ordinals to provide a suitable definition. One can define what a recursive ordinal notation is as a recursively defined *property*. For example we can define a recursive functional as being well founded for the integers as a way of defining the recursive ordinals. To give this definition we set up a notation for integers and TMs such that a symbol in this notation describes either an integer or the Gödel number of a TM that accepts an integer as input and has an element of this notation as output. In set theory we would define a TM as being well founded for the integers if and only if for any infinite sequence of integers we can apply the first integer

to the original TM, the second integer to the output of the original TM, the third integer to the output of the output of the original TM, etc, and in a finite number of steps we will get the notation for an integer and not another TM as output.

Without quantifying over the reals we can define this a property of TMs $\mathcal{F}(n)$. We will let $\mathcal{J}(n)$ be true if n denotes an integer and otherwise $\mathcal{J}_n(x)$ denote output of the TM encoded by n applied to input x . In the following n is any element of the notation.

$$\mathcal{F}(n) \equiv \mathcal{J}(n) \vee \forall_{\omega} x [\mathcal{J}(\mathcal{J}_n(x)) \vee \mathcal{F}(\mathcal{J}_n(x))]$$

$$\mathcal{F}_{\alpha}(n) \equiv \mathcal{J}(n) \vee \alpha = 0 \wedge \forall_{\omega} x [\mathcal{J}(\mathcal{J}_n(x)) \vee \mathcal{F}(\mathcal{J}_n(x))]$$

This recursive definition does not exclude infinite descending chains but it will only allow us to build up structures that are well founded in the set theory sense. This is somewhat analogous to what happens in conventional ZF. We can define the set of all recursive ordinal notations but there is no general way to determine what elements belong to that set. The difference is that we do not speak of the set as if it were a completed entity. We only speak of properties of TMs.

Of course it is useful to talk about the set of all recursive ordinals and I am not advocating that we abandon thinking or doing mathematics in that way. I am advocating that we think of these sets as defined by *properties of TM's*. We should draw a line between the structures in ZF that can be interpreted in this way and those which can only be interpreted as properties of a formal system. The purpose of drawing this line is not to weaken but to strengthen mathematics. Mathematics that goes past this line I call *shadow mathematics*. It is not about objective properties of TMs but denotes ways in which we can extend an existing formal system to extend those properties. Recognizing where this distinction lies and focusing on extending mathematics at that critical point is essential to producing strong extensions. Working in shadow mathematics is a bit like iterating the ω consistency of a system. You can always play that game but it is always a weak generalization compared to what is possible through a deeper understanding

of the system. As we shall see, the concept of an uncountable ordinal bypasses the generalizations of the concept of an ordinal that are necessary to extend the concept within a recursive functional hierarchy. Loosely speaking, if a statement refers to a recursively enumerable collection of events, then it is a statement about a recursive process in a potentially infinite universe and is objectively true or false. Statements, such as the continuum hypothesis, that cannot be so interpreted, are not objectively true or false. They are part of shadow mathematics. They can give insight into good ways to extend a system but they are not themselves valid extensions.

The seemingly weaker approach of actually constructing the recursive functionals at each level in the hierarchy will turn out to be more powerful. When we reach the level in this functional hierarchy where induction up to an ordinal is exhausted we need different generalizations for induction and these will eventually lead us outside of what is definable in ZF in a much more powerful way than large cardinal axioms or other conventional ways of extending ZF. The latter are a way of saying that an object exists that is in some sense inaccessible through certain operations. The functional hierarchy approach does something much more powerful. It allows us to define new kinds of abstractions and new kinds of induction on those abstractions.

All the ordinals in our hierarchy must have a recursive ordering relationship or we cannot use them in recursive functionals. This implies that there is a recursive ordinal that is the limit of everything we can define. We get around this by making our definitions expandable so we can add new functionals. For every recursive ordinal there must be some expansion that includes that ordinal and that expansion must itself be expandable to all larger recursive ordinals.

C.1 Concepts

There are three primary concepts in the initial hierarchy we develop. The first is the notion of a well founded recursive functional hierarchy of typed objects. A functional in the hierarchy accepts objects of a given type as input and outputs objects of a given type.

There is a recursive algorithm to determine if an object in the hierarchy is an allowed input. If we keep applying valid inputs to any member of the hierarchy we will in a finite number of steps get an output which is notation for an integer and not a functional.

The next concept is iterating up to a given ordering. This is far more involved in a recursive functional hierarchy than induction up to an ordinal is in ZF. In set theory one only needs to define what one does at 0, successor and limit ordinals to perform induction. In the recursive functional hierarchy there is a hierarchy or limit types for ordinals. The type of limit ordinal can be that of the integers, notations for recursive ordinals, ordinals that are definable by functionals well founded for notations for recursive ordinals, etc.

The third concept is that of using any ordering relationship we define to iterate the notion of well founded hierarchy up to that 'ordinal'.

The set theory approach to extending such a hierarchy is to posit the existence of structures that are inaccessible through certain kinds of operations. While we can do this it is a weak way to extend this hierarchy. A stronger way to extend the hierarchy involves new *kinds* of abstraction and iteration on those structures. The nature of this abstraction cannot be understood without working out the details of the recursive functional hierarchy. It is precisely these details that are washed away with the seeming more powerful but actually much weaker approach of conventional set theory.

C.2 Ordinals in a recursive hierarchy

In set theory there is only induction up to an ordinal. One cannot use inductions up to an ordinal to *recursively construct* a new object. One can perform recursive operations on a functional that is a notation for an ordinal to construct a new functional that represents the result of set theory style induction. The sequences of parameters this new functional accepts and the operations performed on those parameters can mirror the structure obtained from set theory style induction.

For example one can recursively define an ordinal function on recursive ordinal notations, $f(x) = x + \omega$, as a functional that outputs a notation for the ω successor of x . The notation generated is a function on the integers that outputs a notation for the n th successor to x for integer input n .

In set theory one can do the same operation at any limit ordinal because the elements in that limit (and their union) are each sets. In a recursive functional hierarchy the type of limit determines the nature of the operation. If the ordinal type is that of the integers then one typically constructs a functional on the integers that iterates some process up to the value of the integer parameter. If the type is that of recursive ordinal then one defines a functional that does different operations for 0, a successor ordinal and a limit ordinal. The next level in the hierarchy of limit types is that of a functional well founded for notations for recursive ordinals. Recursive ordinals can be thought of as providing schemes for constructing functions on the integers through integration and diagonalization. Functional well founded for recursive ordinals provide similar schemes for iterating the construction of recursive ordinals. A limit type of all such objects is typically a process for doing induction on such schemes.

We can iterate the notion of a limit type that is well founded for a lower limit type. We can iterate this up to any ordinal structure we define. At this point we have exhausted the notion of ordinal in recursive functional hierarchies. The next step in set theory is all ordinals definable in this way but that set is the set of all countable ordinals. This would take us outside the domain of recursive processes in a potentially infinite universe. Instead we need to generalize the notion of ordinal to allow induction on all objects of this class without resorting to uncountable structures. It is at this point that we *must* part from set theory style definitions.

C.3 C++ constructs

To construct this hierarchy the C++ construct of `class` and member function is particularly useful. To formalize the construction it is helpful to be able to talk about such constructs directly. We can

regard a class definition as a TM program that accepts multiple parameters. The first parameter is the instance of the class. This is values for all the fixed data elements in the class. The second parameter is a class member function name. The remaining parameters, which there can be any finite number of, are the parameters of the member function. When a complete set of parameters are specified an output object is computed (or the member function runs forever). It is possible to define a `void` member function that has no output but, for example changes its input. We will not use such constructs in the formalization. There can be different interpretations of a member functions output but the interpretation is fixed for each specific member function.

Classes can be derived from base classes and member functions can be overloaded. When a class is derived all the member functions of the base class are inherited and can be called from the derived class. A virtual function can be redefined in a derived class. If the virtual function is called then the version from the highest level class that defines the function will be invoked. The lower level member functions with the same name will be ignored.

We will denote the invocation of a class member function as:

```
class_instance->class_name::member_functions(parameters).
```

This is standard C++ notation except the `'class_name::'` part is optional and seldom used. The compiler can find the class name by looking up the `class_instance` in a table. (One must use this notation if one wants to call a virtual function for a specific class and ignore any higher level virtual functions with the same name.) We will also leave out the class name whenever the class is obvious.

There are several other relevant aspects of C++ syntax. Every object in C++ has a type. A class instance object has the type of the class. There are several built in types such as integer (`int`) and floating point (`float`). The only type we will use aside from classes we define is `very_long`. This will represent an arbitrarily large integer. To implement it with arbitrary size require a class `very_long`. Here we will define it to be a 64 bit integer. If necessary an appropriate class can be defined to fully implement it.

Member functions names can be overloaded. There can be two member functions with the same name but with different parameters. The function called is determined by the number and types

of parameters. A member function can be a standard operator in the language. One can define a member function `operator+`. This describes what operation to perform whenever two class objects, `a` and `b` appear in an expression such as `a+b`. (By using this feature we can define a class `very_long` that does arbitrary size arithmetic for all the standard arithmetic operations.) There can be a member function `operator()`. This defines the routine to call whenever one writes `class_object(parameter)`.

C.4 Sets versus recursive functionals

We can fully model the structure of any recursive ordinal with a functional on the integers. That is a functional that accepts integers as input and has as output a notation that represents either an integer or a functional on the integers. We need the ordering of elements in the domain of a functional to match the ordering of elements in the range. That is if $a < b$ we want $f(a) < f(b)$. To make this possible we must allow two kinds of elements in the range of the functional. The first represents a subset and the second a union of elements each of which *individually* represents a subset. For example the ordinal $\omega \times \omega$ is represented by a functional f on the integers such that $f(1) = \omega$, $f(2) = 2 \times \omega$, etc. However $f(0)$ does not represent a set but an infinite union of sets (all the integers) that are each members of the ordinal represented just as $f(1)$ is a member.

It does not matter how one builds up the ordinal hierarchy in set theory because each ordinal is the union of all smaller ordinals plus the set containing this union. The set for a particular ordinal will be the same no matter how it is constructed. The structure we must build is more complex and has additional constraints. We want to make all operations on these structures effective and thus we must insure that we can have a recursive process to determine if any two ordinal notations are identical and if not which is larger.

It might seem that this restricts us to recursive ordinals. However we can define an expandable recursive notation. The notation is recursive and has a recursive expansion for every recursive ordinal. The natural way to define this property requires self refer-

encing statements of a type not allowed in ZF. We can define the property in terms of sequences of sets but this is awkward. As we expand the hierarchy we need to generalize this definition in complex ways and the corresponding set theory definition becomes more awkward.

C.5 A recursive functional hierarchy

Definition 3 *A notational hierarchy is a hierarchy of 0 or more derived classes, a single base class `Functional` and a recursively enumerable set of instances of those classes.*

Definition 4 *A notation is an instance of class `Functional`.*

Definition 5 *A notational hierarchy h_2 is an extension of a notational hierarchy h_1 if h_1 is a subset of h_2 .*

The notation for an integer must, like everything else in the hierarchy, be an instance of the class `Functional`. We get the integer value from the notation by calling a member function, `int_value`. If a `Functional` is not a notation for an integer this member returns `-1`.

Definition 6 *An notation for an integer has the following properties.*

1. *There is a member function `valid_parameter(f)` that 0 for any functional `f` in the hierarchy.*
2. *There is a member function `int_value()` that returns the integer represented.*

Class `Functional` represents arbitrary structures. Integers are treated as a special case with member function `int_value`. Integers are the finite ordinals. For infinite ordinals we need more structure and this is best handled with a derived class `Ordinal`. To determine if an arbitrary functional is an ordinal we will have a member function `ordinal_value`. This will return a null pointer if the `Functional` does not represent an ordinal. If it does represent an ordinal it returns a pointer to itself that can then access

member functions specific to ordinals. Functionals that represent integers must be derived from an instance of class `Ordinal` and return a pointer from `ordinal_value`.

Ordinals must be recursively well ordered. There is a member function `order` of `Ordinal` such that for any two ordinal notations `a` and `b` `a->Ordinal::order(b)` returns -1, 0 or 1 if `a` is less than, equal to or greater than `b`. Since `b` may be from an expansion of the notational hierarchy that `a` knows nothing about the `order` function for `a` may return `-b->order(a)`. To know whether this is needed there is an `Ordinal` member function `level` that returns an integer value representing the level in an expanded notational hierarchy. The `order` function of the ordinal with the largest value of `level` must be used. The orderings defined in a particular expansion will always have a fixed recursive ordinal as a limit. This limit does not hold for sequences of expansions.

As mentioned before, in contrast with ZF, there are many types of limit ordinals. Limits are characterized by the type of parameter they allow. The lowest level is the integers. The next level is notations for recursive ordinals (which can be defined as those ordinals represented by structures well founded for the integers). The next ω levels correspond to structures definable by well foundedness on the previous level of object. We can integrate this up to any recursive ordinal. We want to be able to iterate it up to any recursive notation we can define.

Definition 7 *A notation `f` is an ordinal notation if `f->ordinal_value` returns `f` as a pointer to an instance of class `Ordinal`. `Ordinal` must be derived from class `Functional`. `Ordinal` has member functions `order`, `level`, `predecessor` and `limit_type`. as described above.*

C.6 Functional hierarchy axioms

We will give the axioms defining the objects that exist in our system. We will follow each axiom with the C++ code to implement the axiom or construct the objects that it defines from other objects. These axioms will use recursive definitions that are not in general

valid in ZF. At each stage at which we can, we will give a definition in ZF of the objects intended at that stage. The ZF definition will exhaustively define the objects that meet a given recursive definition. In particular the ZF definition will disallow any infinite descending functional evaluations. In the recursive definitions these are not explicitly prohibited (to do requires quantification over the reals) but only objects that satisfy this can be built up from the definitions. This is similar to ZF where well founded sets are not explicitly excluded (the axiom of regularity is generally not considered a part of ZF) but only well founded sets can be constructed.

In this formulation objects from C++ class `Functional` take the place of sets in ZF. Everything, well almost everything, is an instance of this class. Of course we have the structure of C++ as a preexisting framework for the structures we define. It is also convenient to treat integers as a special case. A `Functional` that represents an integer has a member function that returns the C++ notion of an integer as a `very_long` object. This can be defined as a standard C++ integer object or as a class object that implements arbitrarily long integers.

The axioms loosely follow the axioms for ZF defined in Appendix A.2 on page 79. The power set axiom is replaced by the well foundedness axiom. The replacement axiom is replaced by the combination of the well foundedness axiom and the induction axiom.

C.6.1 Axiom of integers

Because we treat the integers we have axiom for integers instead of the axiom of the empty set. of the integers.

Axiom 1 *Axiom of integers. There is a functional that represent every integer.*

This may not seem satisfactory since it assumes the integers already exist. However we are assuming the framework of C++ and they do exist in that framework. All of these axioms are informal. If you want to do a fully formal system you need to start with first order arithmetic or some other system strong enough to embed a universal TM and then define the constructs in C++ that we use

in that system. The code for this is trivial. You only need to have an integer object as a member of class `Functional` and return the value of that object for member function `int_value`. To write the code for this we need to define the class `Functional`. The full class definition is in Figure C.1.

Many of the member functions have already been described. We will describe here some additional functions that are generic. Others will member functions and the code for member functions that are not a part of the class definition will be included where and if they are needed. Some functions are for housekeeping or other purposes and will not be described here. They included because this is compiled C++ code which is intended to be used and has been to a limited degree tested. The source code is available at TO BE DETERMINED.

We use virtual C++ functions to make this classes open ended. A `Functional` maps `Functional`'s to other `Functional`'s through a virtual function `operator()`. By deriving classes from base class `Functional` and by redefining the virtual member functions, such as `operator()`, we have a completely open structure with flexibility similar to that of a 'set' in set theory. Because every instance of a `Functional` must be implemented as an effective procedure, we need to know if a given parameter is valid. This is determined by another virtual function `valid_parameter`. This is the base class for all functionals no matter what structure they represent. As new levels of the hierarchy are defined the virtual functions in `Functional` must be defined in derived classes to work with all previously defined code.

If an integer is being represented then `the_limit` is the integer being represented otherwise it is -1. Member `union_depth` is used to take the union of an infinite set. Virtual member function `clone_base` is used to create a copy of this object including all the derived classes that this object may also be an instance of. This function is never called directly but only by function `clone`.

C.6.2 Axiom of finite iteration

In this and the following axioms on iteration I introduce structures to perform iteration up to an integer value (a standard loop), a recursive ordinal and larger ordinals defined by a well founded functional hierarchy. In contrast to ordinal iteration in set theory we are always dealing with a process that produces a result in a finite number of operations. The output may be a functional and its range can represent an infinite set. This is not that different than set theory. All sets constructible in a formal system are constructible in a finite number of operations in terms of applying the axioms of the theory to prove the existence of the set. We do not and cannot construct the sets in the domain of the recursive functionals we define but we can always construct the functional itself.

```

class Functional: public FunctionalEvaluate {
    very_long union_depth ;
public:
    Functional(very_long n):FunctionalEvaluate(n), union_depth(0){}
    Functional(Functional ** const m, very_long u=0):
        FunctionalEvaluate(m),union_depth(u){}
    Functional(very_long * iter, very_long depth=0,
        very_long exp_depth=0, Functional* func=0):
        FunctionalEvaluate(this,iter,depth,exp_depth,func),
        union_depth(0){}
    virtual Functional * clone_base() {return new Functional(*this);}
    Functional * clone() {return clone_base();}
    Functional * increase_union_depth(very_long inc)
        {union_depth+= inc; return this;}
    Functional * clone(very_long depth)
        {return clone_base()->increase_union_depth(depth);}

    virtual Ordinal * ordinal() {return 0 ;}

    static Functional * set_union(Functional * c);
    // union

    // Functional * convert_to_int(very_long n);
    Functional * make_invalid() {delete_data(); return this;}
};

```

Figure C.1: Functional base class

Appendix D

A discrete model for physics

There are many reasons to suspect that at the most fundamental level nature is discrete. These go way beyond the philosophical arguments I have already described for the nonexistence of infinite or continuous structures. Perhaps the most compelling is finiteness in quantum mechanics. The information content of any finite region containing a finite amount of energy is finite. It would be an extraordinary extravagance to use continuous structures to embed finite information. Everything we know about nature leads to the conclusion that she abhors such extravagance.

If one assumes nature at the most fundamental level is both discrete and simple one is led to consider models in which space and time are described by discrete points connected in a topological structure such as a grid. Although we visualize this as a structure in physical space it is important to understand that the topological arrangement of points does not exist in space but is the basis for space. Spatial relations are determined by the topological connection of points and not vice versa. Further the metric or distance function that we observe is not just a function of these topological connections. If relativity is even approximately true the distance we observe between two points must be a function of this topological structure and the physical state defined on the topological structure.

If that physical state is discrete it can be described by a set of integers or a single integer at each point. The laws of physics would then be reducible to mathematical rules for describing a fu-

ture state in terms of earlier states. If these rules are to be simple they must be local. That is the state of a point at some time is determined by the state at a small number of previous time steps and at a small number of previous locations that are in the immediate neighborhood of this point. Models of this type are discrete field models. They are the discrete analogue of the continuous field models that Einstein spent most of his life trying to reduce physics to.

If one suspects that physics can be reduced to a discrete field model then one is led almost inevitably to a set of rules for describing how that field evolves. The wave equation is universal in physics describing both the evolution of the electromagnetic field and quantum mechanical wave function for particles with no rest mass. Discretized rules for describing state evolution that approximates the classical wave function are known as discretized finite difference approximations to the wave equation. The remarkable thing about discretized finite difference approximations to the wave equation is that they may account for all of physics including all the properties of all the fundamental particles. A model that one can easily write down on a half a sheet of paper may be sufficient to explain all of physics.

D.1 A simple toy discrete model

One way to get some insight into discretized finite difference equations is with simple toy cases. The simplest model of this class is one dimensional in time and has a single point in space. This defines a sequence of integers, x_n , with the entire sequence being determined by the first two elements. The discretized finite difference equation is $x_n = T(\alpha x_{n-1}) - x_{n-2}$, where α is a constant that determines the frequency of the solution and $T(x)$ truncates x towards 0, i.e., $T(1.7) = 1$ and $T(-4.8) = -4$. The exact solution to the difference equation for $-2 \leq \alpha \leq 2$ without the truncation operation is $A \cos(\omega n)$, where $\omega = \arccos(\alpha/2)$ and $A = \sqrt{x_0^2 - ((x_0 \cos(t) - x_1)/\sin(t))^2}$.

The discretized equation approximates the continuous one but has some additional interesting properties. The discretization forces

the sequence of values to repeat after some finite initial segment provided it does not diverge. There are only a finite number of pairs of integers below any fixed upper limit and thus after some time one of these pairs must be repeated. Since all subsequent values are determined by any two adjacent values in the sequence repetition of such a pair means the sequence is looping. Because the discretized equation preserves the exact time symmetry of the differential equation any repeat loop must include the initial state. We can reverse the sequence by reversing the order of two successive values. This will generate the values in the loop in reverse order. Thus the initial state must be in the loop.

For a given value of α the equation partitions all pairs of integers into disjoint sets that are in the solution for some set of initial values. One interesting result is the combinatorial complexity introduced by discretization. In trying 186 similar initial conditions with a fixed value of α the repeat length ranges from 2,880 to 148,092 samples. Table D.1 on page 110 gives these lengths. The program that generated this table is available as TO BE DETERMINED.

D.2 Problems with a discrete model

Discretized finite difference equation models have two serious problems. To scientists and mathematicians who think they are working with continuous or completed infinite structures these models seem ugly and arbitrary. This is a matter of taste and familiarity. Most applaud the digital revolution when it brings flawless audio recordings, high definition television or cheap and powerful computing. However many mathematicians and scientists prefer the illusion that the work *they* do transcends the 'simple minded' structures of discrete bits. One theme of this book is how limiting that illusion is. Such things change slowly. Tastes will change if the approaches I advocate lead to the results I expect but that will happen only long after those results have been digested.

A far more important problem is the difficulty in deducing detailed macroscopic predictions from these models. The intellectual thinking based research model that has evolved in the last few centuries does not consider mere ideas or intuitive arguments to

Difference equation: $x_n = \alpha x_{n-1} - x_{n-2}$

Exact solution: $A \cos(1.42023n)$

x_0	x_1					
	1005	1006	1007	1008	1009	1010
10010	51632	114116	114116	22602	105330	51632
10011	33830	89448	20682	105330	114116	82060
10012	114116	20682	131104	29176	28216	12741
10013	86714	34790	72460	33830	26442	19315
10014	22602	48084	78366	85900	131104	26442
10015	43430	33830	118770	16028	19315	148092
10016	89448	48491	118770	90262	51632	19315
10017	48084	19315	48084	48084	68620	123424
10018	115222	85900	86714	22602	90262	89448
10019	43430	131104	114116	90262	148092	90262
10020	41510	85900	85900	24522	86714	33830
10021	48084	23562	37670	105622	68620	89448
10022	131104	23562	33830	7534	24522	86714
10023	105622	118770	108356	89448	7534	118770
10024	66992	89448	148092	7941	68620	19315
10025	20275	19315	123424	45350	108356	5207
10026	123424	20275	45350	89448	115222	20682
10027	68620	68620	37670	16028	108356	48491
10028	108356	89448	108356	20682	68620	123424
10029	90262	78366	148092	66992	16028	48491
10030	86714	23562	78366	108356	66992	48084
10031	89448	68620	86714	45350	66992	148092
10032	51632	115222	4247	16028	148092	89448
10033	118770	2880	45350	16028	45350	7534
10034	16028	16028	16028	51632	2880	85900
10035	23562	2880	3287	20682	27256	31388
10036	29176	45350	16028	41510	105622	19315
10037	123424	78366	11374	123424	115222	31388
10038	41510	26442	20682	23562	66992	21642
10039	20682	85900	21642	51632	51632	11374
10040	72460	78366	29176	148092	21642	33976

Table D.1: FDE repeat length for similar initial conditions

support them as an adequate basis for hard science. New science starts out that way but it must be converted to a mathematical theory with testable predictions before it is considered proper science. In recent years the physics community has relaxed half of this requirement but it is the wrong half. Much research in foundations in quantum mechanics such as quantum gravity is far from producing models with experimentally testable consequences. That is no longer seen as essential as long as the models are mathematically sophisticated. This puts the existing mathematical formulation ahead of experiments by implying we can base new theory on mathematical extensions of the existing theory without experimental tests. This is nonsense. It is an example of pushing an intellectual approach to problem solving way past the point that it is productive. The models in this chapter are the reverse. They have experimentally testable consequences but these predictions are based on simple mathematics and intuitive arguments that are far from providing a substantial mathematical development of the model.

Nature is under no obligation to be based on laws that are discoverable by any particular research model. Indeed if nature is simple at the most fundamental level then those simple laws are almost certainly such that it is extremely difficult to develop a complete theory from them. If the laws were simple and the derivation of a theory straightforward then the laws would have been discovered and the theory developed long ago.

The theory presented here is in a primitive state of development. It is important because it shows how a more complete local theory is possible. This can motivate effective tests of Bell's inequality and help to define what constitutes an effective test. If and when such tests show quantum mechanics is false, there will be an intense interest in developing a more complete and correct theory. I do not think the existing research model will be able to produce such a theory. We will have to work for a longer time and collectively with ideas like those presented here that are still at a primarily intuitive state of development. If my models are the correct approach we will need to create new branches of mathematics to describe the large scale properties of discretized finite difference equations. We must do this with primarily intuitive arguments for why that math-

ematics is likely to be important. Both the development of this mathematics and experimental data from test of Bell's inequality and follow on experiments may be essential to creating a detailed mathematical theory describing macroscopic consequences. This development is likely to be more far more difficult than the development of quantum mechanics.

D.3 The history of Bell's inequality

Tests of Bell's inequality are essential to determining if my approach is correct and to motivating the research need to develop my approach. Thus it is useful to briefly review the history that led to Bell's inequality.

In the early 1930's von Neumann published a paper that claimed to prove that no more complete theory could be consistent with the predictions of quantum mechanics[33, 34]. Two years later Einstein, Podolsky and Rosen, in a classical paper frequently referred to today as EPR, argued that quantum mechanics is an incomplete theory because properties that are conserved absolutely such as momentum must have some objective reality to them and quantum mechanics does not model this objective reality[15]. Einstein later argued in a famous series of debates with Bohr that the that there must be additional hidden variables that provide a more complete description of a quantum mechanical system. In the early 1950's Bohm published what he called a "hidden" variables theory' that was nonlocal but deterministic and consistent with the predictions of quantum mechanics[6]. This suggested there was something wrong with von Neumann's proof. At the time Bohm felt that his theory was different than quantum mechanics at some point and thus escaped von Neumann's proof but this apparently is not true and it is the nonlocal nature of Bohm's theory that allows it to reproduce the predictions of quantum mechanics in a deterministic model.

In the 1960's Bell showed that von Neumann's assumptions were too restrictive[4]. Bell felt it was not possible to prove the general result von Neumann claimed and started looking for additional constraints that might allow one to distinguish between quantum

mechanics and hidden variable theories. Influenced by Bohm's work, he developed an inequality that the statistics of events with a space-like separation must obey in any *local* hidden variables theory. (Events are space-like separated if they are close enough in time and far enough apart in space so that no signal traveling at the speed of light can go from one to the other.)

Bell proved quantum mechanics predicts this inequality is violated[3]. In the 1970's Eberhard derived Bell's result without reference to local hidden variable theory, it applies to all local theories[12, 13]. Eberhard also showed that the nonlocal effects quantum mechanics predicts cannot be used for superluminal communication. Eberhard assumed a statistical law known as *contrafactual definiteness* in his derivation. That is he assumed that he could average over all the *possible* outcomes of a *single* experiment. Previously Stapp had argued that Bell's inequality was evidence that this property does not hold for quantum mechanical effects[30]. This assumption is a standard one in statistical analysis. Some argue that the uncertainty principle is a reason for doubting this principle for quantum effects. How can there be a definite outcome for an experiment not performed in quantum mechanics? The reason one can make such arguments is the claim that probabilities are irreducible in quantum mechanics. There is no mathematics of irreducible probabilities and reason to doubt that one can develop such mathematics. Thus one can argue that probabilities in quantum mechanics are something different than in classical mechanics and completely outside of any known mathematics. In theory one can ascribe any property one wants to probabilities in quantum mechanics. Thus, for example, Youssef has argued that probabilities in quantum mechanics are complex and not real valued[40].

I doubt all of this. I think the difference between classical and quantum mechanics is in the conceptual framework of the theories as discussed in Section D.10 on page 123 and not in basic principles of mathematics or statistics.

D.4 Tests of Bell's inequality

At the time Bell's result first became known, the experimental record was reviewed to see if any known results provided evidence against locality. None did. Thus an effort began to develop tests of Bell's inequality. A series of experiments conducted by Aspect ended with one in which polarizer angles were changed while the photons were 'in flight'[1]. This was widely regarded at the time as being a reasonably conclusive experiment confirming the predictions of quantum mechanics. Aspect's experiment had the arrangement shown in Figure D.1 on page 126.

Pairs of photons in a singlet state are emitted by the source in opposite directions. Each traverses a polarizer and is detected. Quantum mechanics predicts a relationship between the angle of the two polarizers and the probability for detecting both photons from a singlet state pair that no local theory could reproduce *provided* the delay between when the polarizer angles change and the probability of joint detections changes is less time than it takes light to travel from either polarizer to the detector most distant from it. Three years later Franson showed that Aspect's experiment to test Bell's inequality did not rule out local realistic theories with delayed determinism[17].

Discretized finite difference equation models possess delayed determinism in the sense that a system can begin to converge to a state and then reverse the process as discussed in Section D.9 on page 120. There is no definite time at which a state is determined absolutely. A state is only determined statistically as there is always some nonzero probability that it can be reversed. Discretized finite difference equation models are not hidden variables theories in the sense that they are not theories of particles plus hidden variables. They are theories of 'hidden' distributed information stored holographic like throughout a space time region. This information cannot be uniquely associated with individual particles although it determines the results observed in particle interactions. The classical parameters of an interaction are determined as *focal points* of continuous nonlinear changes in the wave function and not as discrete events. In addition to not violating Bell's inequality this class of theories can in principle be distinguished from standard quan-

tum mechanics by other experiments.

Franson's notion of delayed determinism i.e. that an event may not be determined until some time after it has been completed, may seem strange and unrealistic. However there is no objective definition of event in quantum mechanics. The unobserved microscopic events that Franson discusses (such as the emission of a photon by an excited atom) are hypothetical. It is a mistake to assume that such events occur as macroscopic events do. Quantum mechanics only allows us to compute the probabilities of making observations given certain initial conditions. What happens between the time we set up the initial conditions and make an observation is the *terra incognita* of quantum mechanics. We cannot base the timing in a test of Bell's inequality on the hypothetical times of hypothetical events.

Franson's objections to Aspect's experiment showed that there is no *objective* criteria in the formalism of the existing theory for computing the timing in an experimental test of Bell's inequality. One way to understand this is through the thought experiment of Schrödinger's cat[29] as discussed in Section 4 on page 32. There is nothing in the formalism of quantum mechanics that allows us to know when macroscopic events are irreversibly determined. That question is left to interpretations which for the most part are metaphysical and not subject to experimental tests. Thus there is no way to decide among them. This problem applies not only to tests of Bell's inequality but to any experiment that asks questions about the timing of causal sequences of *macroscopic* events.

If the timing cannot be derived from the formalism of quantum mechanics or from an interpretation of the theory then it must be derived from a competing theory. Developing such alternatives, even if extremely speculative, is a critical element in designing tests of Bell's inequality. The timing constraints I describe in Chapter D.12 apply to a broad class of alternative theories and not just the class of models I advocate. These timing constraints are often assumed by experimenters perhaps without fully realizing that they cannot be derived from the formalism of the existing theory.

A recent analysis which claims to describe how to close all the loopholes in tests of Bell's inequality[26] is incomplete in its analysis of the timing issues. The authors state on page 3210: "To close

this loophole, the analyzer's settings should be changed *after* the correlated pair has left the source." There is no way to know when the pair has left the source unless one detects them at that point which makes the experiment impossible. The speed of the process that generate the photons is only relevant if there is a common trigger for that process and the changing of the polarizer angles. Perhaps this is what the authors are suggesting. The timing can only involve *macroscopic* events such as setting the polarizers or *macroscopic effects* from detecting the photons. The basis for determining the times of these events must come from a competing theory. The authors do not discuss this or the need to base timing on purely macroscopic events. In Section D.12 we describe what must be done to address the timing issue in practical experiments.

D.5 Discretizing the wave equation

The simplest model for a local deterministic physical theory is a field function i.e. a function defined at each space time coordinate whose evolution is determined by the previous field values in the immediate neighborhood. The starting point for any theory like this must be the classical wave equation for that equation is universal in physics describing both electromagnetic effects and the relativistic quantum wave function (Klein Gordon equation) for the photon.

By 'discretized' I mean an equation that is modified to map integers to integers. A modification is required because there is no finite difference approximation to the wave equation that can do this. The universality of the wave function requires that any discrete model for physics approximates this continuous model to extraordinary accuracy. Discretizing the finite difference equation adds a rich combinatorial structure that has a number of properties that suggest quantum mechanical effects. Perhaps the most obvious is that an initial disturbance cannot spread out or diffuse indefinitely as it does with the continuous equation. It must either diverge and fill all of space with increasing energy or break up into independent structures that will continue to move apart, i.e., become quantized.

We describe how to approximate the wave equation with a discretized finite difference equation. Let P be defined at each point in a 4 dimensional grid. To simplify the expression for P_{xyzt} we will adopt the following conventions. Subscripts will be written relative to P_{xyzt} and will be dropped if they are the same as this point. Thus P_{t-1} is at the same position in the previous time step. $P_{x-1,y-1}$ is at the same time step and z coordinate and one position less on both the x and y axes.

The wave equation is approximated by the difference equation:

$$P_{t+1} - 2P + P_{t-1} = \alpha(P_{x+1} + P_{x-1} + P_{y+1} + P_{y-1} + P_{z+1} + P_{z-1} - 6P)$$

The difference equation discretizes space and time but not the function defined on this discrete manifold. The simplest approach to discretizing the function values is to constrain them to be integers. This requires either that α be an integer or that some rounding scheme be employed that forces the product involving α to be an integer. The former is not possible since it does not allow for solutions that approximate the differential equation.

D.6 Properties from discretization

From the time symmetry one can conclude that any solution must either diverge or loop through a repeated sequence that includes the initial conditions. The restriction to looping or divergence follows from the discreteness (there are a finite number of states) and causality (each new state is completely determined by the 2 (or N depending on the differencing scheme) previous states. The loop must include the initial state because of time symmetry. At any time one can reverse the sequence of the last 2 (or N) states and the entire history will be repeated in reverse. Thus any loop must include the initial conditions.

The time required for a given system to repeat an exact sequence of states based on the number of possibilities easily makes astronomical numbers appear minute. However if there are only a small number of stable structures and the loops do not need to be exact but only produce states close to a stable attractor then we can get a form of structural conservation law.

For large field values this model can approximate the corresponding differential equation to an arbitrarily high precision. As the intensity decreases with an initial perturbation spreading out in space a limit will be reached when this is no longer possible. Thus something like field quantization exists. Eventually the disturbance will break up into separate structures that move apart from each other. Each of these structures must have enough total energy to maintain structural stability. This may require that they individually continue to approximate the differential equation to high accuracy. Such a process is consistent with quantum mechanics in predicting field quantization. It differs from quantum mechanics in limiting the spatial dispersion of the wave function of a single photon. It suggests that the wave function we use in our calculations models both this physical wave function and our ignorance of the exact location of this physical wave function.

D.7 A unified scalar field

An ambitious goal for this class of models is to unify all the forces and particles in nature using a single scalar field and a simple rule for describing the evolution of that field. The quantum wave function and the electromagnetic field are identical in this model as they are in the Klein Gordon equation for a single photon and the classical electromagnetic field equation.

All energy is electromagnetic. This requires some way to construct neutral matter from an electromagnetic field. The Klein Gordon equation for a particle with rest mass presents an additional problem.

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \nabla^2 \psi - \frac{m^2 c^2 \psi}{\hbar^2}$$

This is the classical wave equation with a new term involving the rest mass of the particle. How can it be derived from the same rule of evolution that approximates the classical wave equation? This may be possible if there is a high carrier frequency near the highest frequencies that can exist in the discrete model. The Schrödinger wave equation for particles with rest mass would represent the average behavior of the physical wave. It would be the equation for a

wave that modulates the high frequency carrier. The carrier itself is not a part of any existing model and would not have significant electromagnetic interactions with ordinary matter because of its high frequency.

Such a model may be able to account for the Klein Gordon equation for a particle with rest mass. A high frequency carrier wave will amplify any truncation effect. Because of this the differential equation that describes the carrier envelope is not necessarily the same as the differential equation that describes the carrier. If the carrier is not detectable by ordinary means then we will only see effects from the envelope of the carrier and not the carrier itself. The minimum time step for the envelope may involve integrating over many carrier cycles. If round off error accumulates during this time in a way that is proportional to the modulation wave amplitude then we will get an equation in the form of the Klein Gordon equation.

The particle mass squared factor in the Klein Gordon equation can be interpreted as establishing an amplitude scale. The discretized wave equation may describe the full evolution of the carrier and the modulating wave that is a solution of the Klein Gordon equation. However, since no effects (except mass and gravity) of the high frequency carrier are detectable with current technology, we only see the effects of the modulating wave. No matter how localized the particle may be it still must have a surrounding field that falls off in amplitude as $1/r^2$. It is this surrounding field that embodies the gravitational field.

If discretization is accomplished by truncating the field values this creates a generalized attractive force. It slows the rate at which a structure diffuses relative to a solution of the corresponding differential equation by a marginal amount. Since the gravitational field is a high frequency electromagnetic field it will alternately act to attract and repel any bit of matter which is also an electromagnetic field. Round off error makes the attraction effect slightly greater and the repulsion slightly less than it is in solutions of the continuous differential equation.

Because everything is electromagnetic in this model special relativity falls out directly. If gravity is a perturbation effect of the electromagnetic force as described it will appear to alter the space time metric and an approximation to general relativity should also

be derivable. It is only the metric and not the space time manifold (lattice of discrete points) that is affected by gravity. Thus there is an absolute frame of reference. True singularities will never occur in this class of models. Instead one will expect new structures will appear at the point where the existing theory predicts mass will collapse to a singularity.

D.8 Symmetry in a fully discrete model

A fully discrete model cannot be completely symmetric as a continuous model can be. There are ways around this like using a random lattice but such models implicitly assume a continuous manifold. In a fully discrete model there must be an absolute frame of reference and preferred directions in that frame related to the graininess of the lattice that defines the space time manifold. One would expect experimental affects from this absolute frame of reference and perhaps such affects have already been observed. It is conceivable that the symmetry breaking that has been observed in weak interactions is a result of our absolute motion against this manifold and not a break down of parity.

D.9 Dynamically stable structures

It is likely that the structures an initial disturbance breaks into will be somewhat analogous to attractors in chaos theory. These attractors will be dynamically stable structures that pass through similar sequences of states even if they are slightly perturbed. Such structures will be transformed to different structures or 'attractors' if they are perturbed sufficiently. These structures have a form of wave-particle duality. They are extended fields that transform as structural units. It is the 'structural integrity' of these 'attractors' that may explain the multi-particle wave function. These structures can physically overlap. In doing so they loose their individual identities. The relationship between the observation of a particle to earlier observations of particles in a multi-particle system does not require any continuity in the existence of these particles. Particles

are not indivisible structures. They are the focal point and mechanism through which the wave function interacts and reveals its presence.

It is plausible to expect such a system will continually be resolving itself into stable structures. Reversibility and absolute time symmetry put constraints on what forms of evolution are possible and what structures can maintain stability. These may be reflected in macroscopic laws like the conservation laws that predict violations of Bell's inequality. Perhaps we get the correlations because there is an enormously complex process of converging to a stable state consistent with these structural conservation laws. It is plausible that at the distances of the existing experiments the most probable way this can be accomplished is through correlations between observations of the singlet state particles.

In this model *isolated* particles are dynamically stable structures. Multi-particle systems involve the complex dynamics of a nonlinear wave function that at times and over limited volumes approximates the behavior of an isolated particle. Since the existing theory only describes the statistical behavior of this wave function it is of limited use in gaining insight into the detailed behavior of this physical wave function.

Consider a particle that emits two photons. In the existing model there is no event of particle emission. There is a wave function that gives the probability of detecting either photon at any distance from the source. Once one of the photons is detected the other is isolated to a comparatively small region. Prior to detecting either photon there is a large uncertainty in the position of both photons. There is even uncertainty as to whether the particle decay occurred and the photons exist. The existing model gives no idea of what is actually happening. It only allows us to compute the probability that we will make certain observations. Some will argue that nothing is happening except what we observe. In the model I am proposing there is an objective process involving the emission of two photons. There is no instant of photon emission. The photons may start to appear many times and be re-absorbed. At some point the process will become irreversible and the photons in the form of two extended wave function structures will move apart.

An observation of either photon localizes both photons in the existing theory. In my theory there are two localized structures but we do not know the location of these structures until an observation is made. For the most part localization effects do not allow discrimination between my proposal and the standard theory because of the way the existing theory models the localization of entangled particles after an observation. However in an experiment in which a single particle can diffuse over an indefinitely large volume there is a difference in the two theories that is in principle experimentally detectable. Standard quantum mechanics puts no limit on the distance over which simultaneous interference effects from a single particle may be observed. There will be an absolute fixed limit to this in the class of theories I am proposing although I cannot quantify what that limit will be.

Perhaps part of what is so confusing in quantum mechanics is that it combines classical probability where new information allows us to 'collapse' our model of reality in accord with an observation and a physical wave function which determines the probability that there will be a physical nonlinear transformation with a focal point at a given location. The existing theory's failure to discriminate between these two dramatically different kinds of probability may be one reason why it *seems* to defy conventional notions of causality.

Whether a particular transformation can complete depends in part on the conservation laws. Unless there is enough energy to support the new structure and unless symmetry and other constraints are met a transformation may start to occur but never complete. One can expect that such incomplete transformations happen and reverse themselves far more frequently than do complete transformations. The transformations that continually start and reverse could be a physical realization of Feynman diagrams that describe all the possible interactions in a system.

A transformation is a process of *converging to stable state* consistent with the conservation laws. The information that determines the outcome of this process includes not only the averaged or smoothed wave function of the existing theory but also the minute details that result from discretization. This additional hidden information is not necessarily tied to the particles involved or to their wave functions in the existing model. It can be anywhere in the light

cone of the transformation process.

D.10 The conceptual framework of physics

It has often been suggested that quantum mechanical experiments produce results that are inconsistent with classical notions of causality. Bell has proven this is true of the mathematics of quantum mechanics but the issue is still an open one with regard to nature. I believe the problem is not with classical ideas of causality or mathematics but with the conceptual framework with which we view experimental results. It is important to deal with this issue explicitly because it is not possible to fully understand the class of models I propose unless one can think about them in an unconventional conceptual framework.

Consider our inability to simultaneously determine a definite position and momentum for a particle. This result is mathematically related to our inability to simultaneously fix a position and frequency for a classical wave. The only wave that has an exact position is an impulse and that is an integral over all frequencies. We do not think that this implies any breakdown in classical notions of causality. The behavior of a classical wave is completely determined just as the behavior of the quantum mechanical wave function is completely determined.

If point like particles do not exist, it makes no more sense to speak of their position than it does to speak of the position of a classical wave. If what we *observe* as position is the focal point of a nonlinear transformation of the wave function then position is a property of this transformation or interaction and not a property of the particle itself. If these transformations result from a process of converging to a stable state consistent with the conservation laws then the information that determines the detailed characteristics of this transformation may be spread out over a substantial region of space and may propagate in ways that are outside of any accepted theory.

Once two particles interact subsequent observations of one particle puts constraints on observations of the other even after the particles and their wave functions have become separated. It is

quantum entanglement in the mathematics of quantum mechanics that is responsible for violations of Bell's inequality and it is the experimental phenomenon of quantum entanglement that makes nature *appear* to be inconsistent with classical causality.

The energy and momentum in a classical wave is distributed throughout the spatial region occupied by the wave. If two classical waves overlap physically there is no clear way to distribute the energy or momentum at a particular point between the two waves. Once the two wave functions for particles in a multi-particle system become entangled how do they become disentangled? The wave function in the existing theory is of limited help if it only represents the average or statistical behavior of the wave function. If observations of the particles involve convergence to a stable state consistent with the conservation laws the detailed behavior of the physical wave function is dramatically different from and far more complex than its average or statistical behavior in the existing model. Certainly 'disentanglement' will occur if the wave functions of two particles become sufficiently separated. At short distances tests of Bell's inequality will reveal time delays that allow the correlations to be determined by information that propagates locally. At sufficiently great distances the correlations will revert to those consistent with a local hidden variables model. It will appear as if the entangled system collapsed spontaneously into two independent systems. This difference between the existing theory and the class of models I suggest is not limited to Bell's inequality. Perhaps there are experimental tests of quantum entanglement that can more easily be conducted over large distances to discriminate between these alternative theories.

D.11 Delayed determinism

Because this model breaks most of the symmetries of the linear finite difference equation the classical conservation laws are not enforced at the local level. There can be a small discrepancy at any single point and these discrepancies can accumulate in a statistically predictable way. However discreteness and absolute time symmetry combine to create a new class of conservation laws. The

information that enforces them does not exist at any given point in space or time and cannot be determined by a classical space time integral. Instead it is embedded in the *detailed* structure of the state and insures that the same or similar sequence of states will be repeated. The local violations of the conservation laws can never accumulate in a way that would produce irreversible events.

Information throughout the light cone of a transformation puts constraints on what stable states may result. A system may start to converge to two or more stable states but none of these convergences will complete unless one of them is consistent with the conservation laws. The time of the focal point of this process (for example the time when a particle interacts with a detector) and the time when the event is determined, i.e. cannot reverse itself are not the same thing. Since all interactions are reversible in this model the time when an event completes has no *absolute* meaning. It can only be defined statistically, i.e., the time when the probability that the event will be reversed is less than some limit. Quantum mechanics, because it does not model events objectively, cannot be used to compute the probability that an event will be reversed. We must use classical statistical mechanics. As a practical matter we probably need to limit timings to macroscopic measurements where the probability of the measurement being reversed is negligible. In the model we propose statistically irreversible macroscopic events are determined by many reversible microscopic events, i.e. the nonlinear transformations of the wave function. It is important to recognize that use of classical statistical mechanics to define the occurrence of events implies that quantum mechanics is an incomplete theory. It is an assumption consistent with the broad class of theories in which there are *objective microscopic events or processes* that contribute to create macroscopic events.

The distribution of the information that enforces the conservation laws is not modeled by any accepted theory and is not limited by the dispersion of the wave function for the individual particles. This information may be distributed throughout the entire experimental apparatus including both the particle source and the detectors. When quantum entanglement was first discovered there was some thought that it would disappear once the wave function for the entangled particles were spatially separated[18, 7, 9, 8]. As-

pect's earlier experiments[2] tested this. These results indicate that quantum entanglement is not limited by the spatial dispersion of the wave function. In a model like the one we are suggesting the linear evolution of the wave function is only part and by far the simplest part of the picture. Information that enforces the conservation laws through quantum entanglement may evolve in ways that are not remotely close to linear wave function evolution. The only reliable measure of nonlocal quantum entanglement is with direct *macroscopic* measurements of time.

D.12 An effective test of Bell's inequality

Bell's inequality is important because it shows that quantum mechanics predicts macroscopic violations of locality. This can only be tested by suitable *macroscopic* measurements. To discriminate between the class of theories we are proposing one must use statistically irreversible macroscopic events to measure the timing. If the probability of reversal is sufficiently low the events can be treated as if they were absolutely irreversible. If necessary their probability of being reversed can be factored into the experimental analysis. Experimenters often implicitly assume this criteria for the completion of an event even though it cannot be justified in the formalism of quantum mechanics.

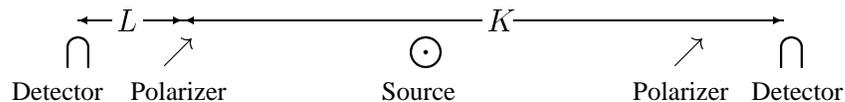


Figure D.1: Typical experiment to test Bell's inequality

Reported experiments generally involve a setup such as that shown in Figure D.1. Quantum mechanics predicts that the correlation between joint detection will change as a function of the polarizer (or other experimental apparatus) settings with a delay given by the time it takes light to travel the distance L . Most experiments are symmetric. L is the distance from either polarizer to

the *closest* detector. Locality demands that a change large enough to violate Bell's inequality can only happen in the time it would take light to travel the longer distance K . K is the distance from either polarizer to the *more distant* detector. To show locality is violated one must show that the delay (D) between when the polarizer settings are changed and the correlations change is short enough that $K/D > C$ where C is the speed of light.

It is technically difficult to directly measure D and none of the reported experiments do this. Indirect arguments about D are all questionable. We have no idea what is happening between the time the excited state was prepared and the two detections occurred. Thus we can make no assumptions about what is happening microscopically. This is true both because quantum mechanics is silent on what is happening and because these experiments are testing the correctness of quantum mechanics itself.

To directly measure D requires that one have a high rate of singlet state events or a common trigger that controls these events and the change in polarizer angles. If this condition is not met the delay we measure will be dominated by the uncertainty in when a singlet state event occurs. After we change the parameter settings the average delay we observe will be $D + .5/r$ where r is the rate of singlet state events and D is the delay we want to measure. If $1/r \gg D$ it will be impossible to accurately measure D . Typical experiments involve distances of a few meters. This correspond to expected values of $D \approx 10$ ns. if locality holds and $D < 1$ ns. if quantum mechanics is correct. A high rate of singlet state events or a precise common trigger for singlet state events and changes in polarizer angles is necessary to discriminate between these times.

To show a violation of Bell's inequality one must show the superluminal transmission of information (at least by Shannon's definition of information). One must show that a change in polarizer angles changes the probability of joint detections in less time than it would take light to travel from either detector to the more distant analyzer. For this change to be sufficient to violate Bell's inequality requires that information about *at least one* (we cannot tell which one) polarizer setting influenced the more distant detector. There must be a macroscopic record to claim information has been transferred. It is the time of that record that must be used in

determining if the information transfer was superluminal.

If one can show superluminal information transfer then one has a violation of relativistic locality (ignoring the predeterminism loop-hole) that is independent of the details of the experiment. Any attempt to enumerate and eliminate all loopholes is insufficient because one can never figure out all the ways that nature might out fox you.

It is worth noting that the historical roots of these predictions is the assumption that the wave function changes *instantaneously* when an observation occurs. This assumption has been built into the mathematics of quantum mechanics in a way that creates irreducibly nonlocal operations. Quantum mechanics insists that there is no hidden mechanistic process that enforces the conservation laws. It is this assumption that creates the singlet state entanglement that enforces conservation laws nonlocally as if by magic with no underlying mechanism.

Bibliography

- [1] A. Aspect, J. Dalibard, and G. Roger. Experimental test of bell's inequalities using time varying analyzers. *Physical Review Letters*, 49:1804, 1982.
- [2] A. Aspect, P. Grangier, and G. Roger. *Physical Review Letters*, 47:460, 1981.
- [3] J. S. Bell. On the einstein podolosky rosen paradox. *Physics*, 1:195, 1964.
- [4] J. S. Bell. On the problem of hidden variables in quantum mechanics. *Reviews of Modern Physics*, 38:447, 1966.
- [5] Eric Berne. *Games people play*. Grove Press, 1967.
- [6] David Bohm. A suggested interpretation of the quantum theory in terms of "hidden" variables, i and ii. *Physical Review*, 85:166, 1952.
- [7] David Bohm and Y. Aharobov. *Physical Review*, 108:1070, 1957.
- [8] David Bohm and B. J. Hilley. *Nuovo Cimento*, B35:137, 1976.
- [9] L. de Broglie. *C. R. Acad. Sci.*, 278B:721, 1974.
- [10] Paul P. Budnik Jr. *Techniques for Parallel Computer Design*. PhD thesis, University of Illinois at Urbana Champaign, 1975. UIUCDCS R-75-763.
- [11] Paul J. Cohen. *Set Theory and the Continuum Hypothesis*. W. A. Benjamin Inc., New York, Amsterdam, 1966.

- [12] P. H. Eberhard. Bell's theorem without hidden variables. *Nuovo Cimento*, 38 B:75–80, 1977.
- [13] P. H. Eberhard. Bell's theorem and the different concepts of locality. *Nuovo Cimento*, 46 B:392–419, 1978.
- [14] Albert Einstein. *Journal of the Franklin Institute*, 221, 1936.
- [15] Albert Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical descriptions of physical reality be considered complete? *Physical Review*, 47:777, 1935.
- [16] Solmon Feferman, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort, editors. *Publications 1929-1936*, volume 1 of *Kurt Gödel Collected Works*. Oxford University Press, 1986.
- [17] J. D. Franson. Bell's theorem and delayed determinism. *Physical Review D*, 31:2529, 1985.
- [18] W. H. Furry. *Physical Review*, 49:393, 1936.
- [19] Kurt Gödel. Consistency-proof for the generalized continuum hypothesis. *Proceedings of the National Academy of Science U. S. A.*, 25:220–224, 1939.
- [20] Kurt Gödel. *The consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis with the axioms of set theory*. Princeton University Press, 1958.
- [21] Carl Gustav Jung. *Alchemical Studies*, volume 13 of *The collected works of C. G. Jung*. Princeton University Press, 1968.
- [22] Carl Gustav Jung. *Psychology and Alchemy*, volume 12 of *The collected works of C. G. Jung*. Princeton University Press, 1968.
- [23] Carl Gustav Jung. *Civilization in Transition*, volume 10 of *The collected works of C. G. Jung*. Princeton University Press, 1970.

- [24] Carl Gustav Jung. *Mysterium Coniunctionis*, volume 14 of *The collected works of C. G. Jung*. Princeton University Press, 1970.
- [25] Carl Gustav Jung. *Psychological types*, volume 6 of *The collected works of C. G. Jung*. Princeton University Press, 1971.
- [26] P. G. Kwiat, P. H. Eberhard, A. M. Steinberg, and R. Y. Chiao. Proposal for a loophole-free Bell inequality experiment. *Physical Review A*, 49:3209, 1994.
- [27] Robert S. McNamara. *In retrospect, the tragedy and lessons of Vietnam*. Times Books, 1995.
- [28] Abraham Pais. *Subtle is the Lord*. Oxford University Press, New York, 1982.
- [29] Erwin Schrödinger. *Proceedings of the American Philosophical Society*, 124, 1935.
- [30] Henry Perce Stapp. S-matrix interpretation of quantum theory. *Physical Review D*, 3:1303–1320, 1971.
- [31] unknown. *The Holy Bible King James Version*. 1611.
- [32] Marie-Louise von Franz. *Number and Time*. Northwestern University Press, 1974.
- [33] John von Neumann. *Mathematische Grundlagen der Quantum Mechanik*. Julius Springer-Verlag, Berlin, 1932.
- [34] John von Neumann. *The Mathematical Foundations of Quantum Mechanics*. Princeton University Press, New Jersey, 1955.
- [35] A. N. Whitehead and Russell Bertrand. *Principia Mathematica*, volume 1. Cambridge University Press, 1910.
- [36] A. N. Whitehead and Russell Bertrand. *Principia Mathematica*, volume 2. Cambridge University Press, 1912.
- [37] A. N. Whitehead and Russell Bertrand. *Principia Mathematica*, volume 3. Cambridge University Press, 1913.

- [38] A. N. Whitehead and Russell Bertrand. *Principia Mathematica*, volume 1. Cambridge University Press, second edition, 1925.
- [39] Richard Wilhelm and Cary F. Baynes. *The I Ching or Book of Changes*. Pantheon Books, New York, 1950.
- [40] Saul Youssef. *Modern Physics Letters A*, 28:2571, 1994.

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