All You Really Need To Know About Bonds

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How Much Would You Pay?

- Assumptions: Coupon paid every 6 months, $100 of principal paid at maturity, government guaranteed

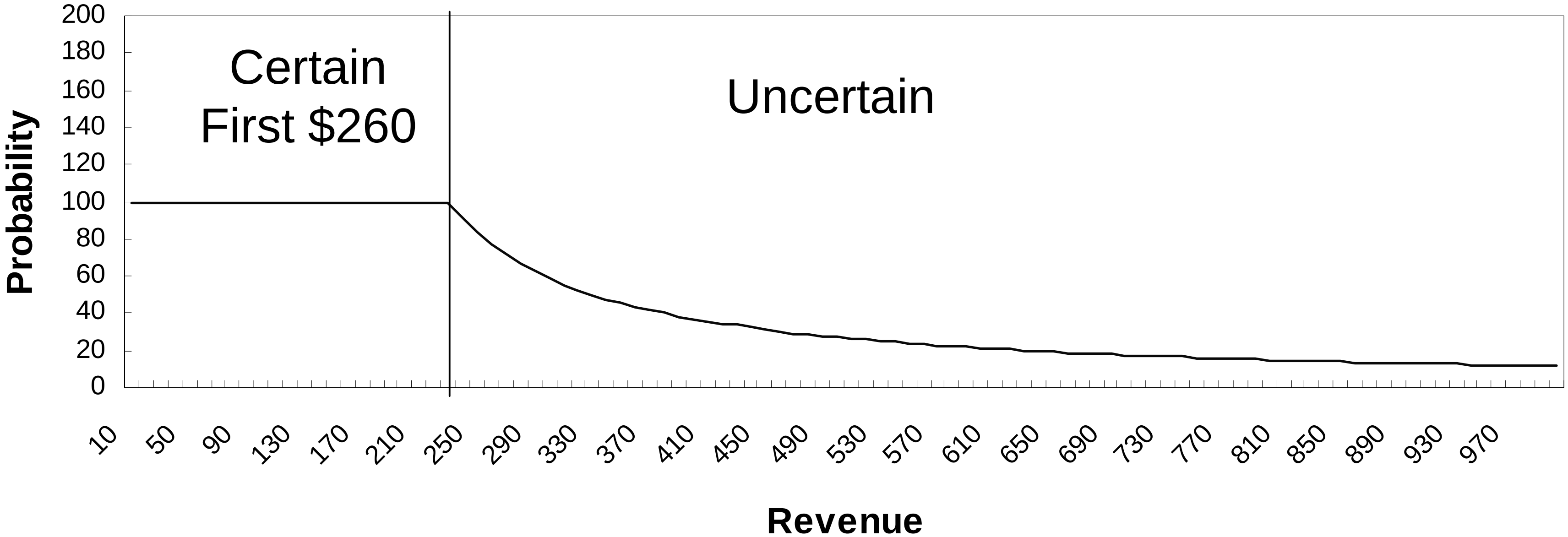
Coupon	Maturity
5.500	02/28/03
5.500	03/31/03
5.750	04/30/03
10.750	05/15/03
5.500	05/31/03

Lets Start Out Slow: What Are Equity and Debt

- Debt is a claim on a **fixed amount** of cashflows in the future
  - A mortgage loan
- Equity is a claim on a **fixed percentage** of cashflows in the future
  - A share of GS
- What's the difference?

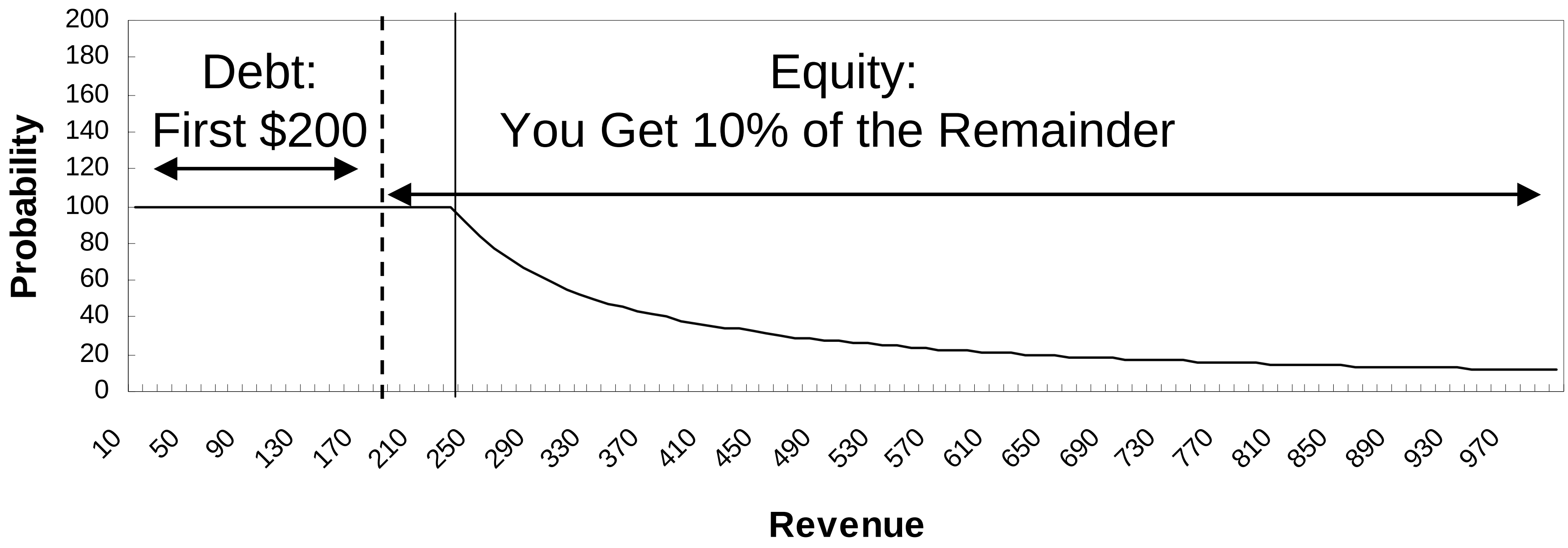
The Difference is Certainty

- All cashflows can be divided into certain and uncertain cashflows

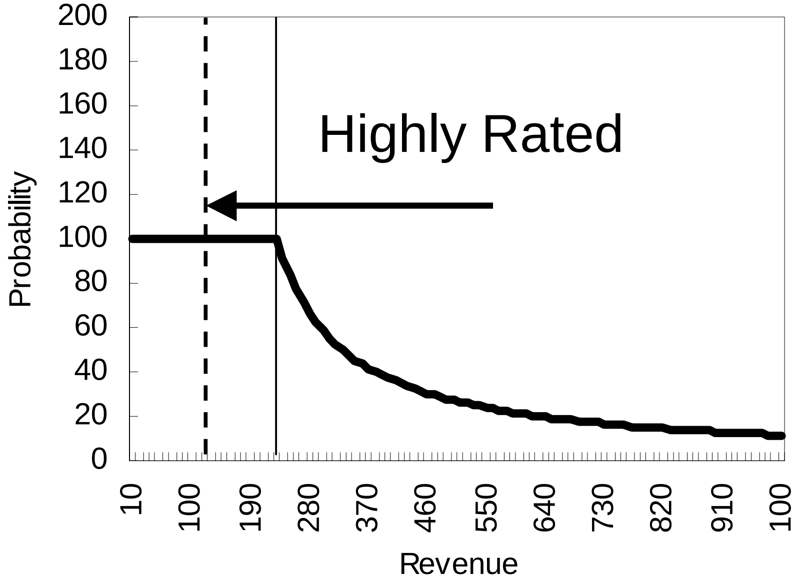


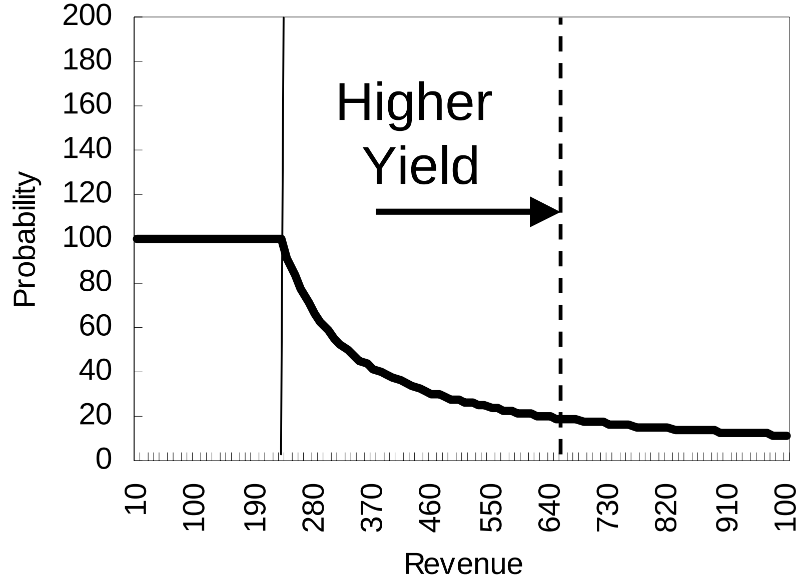
The Debt and Equity Divide Up the Cashflows

- Debt gets the first set of cashflows, equity gets the remainder



Differences Can Be Blurred





The Market Prices: Which Would You Buy?

- Higher coupon, higher price, but is it high enough?
- Higher coupon, longer maturity, lower price, why?

Coupon	Maturity	Price
5.500	02/28/03	98-092
5.500	03/31/03	98-082
5.750	04/30/03	98-266
10.750	05/15/03	111-042
5.500	05/31/03	98-062

How Much is This Cashflow Worth?

You get $6 every year for ten years,
then a lump sum payment of
$100 in ten years

Definition of Debt

- The typical terms of a debt obligation is broken down into two components
- You receive **principal** in N years
- And every year until then you receive a **coupon** payment
  - This coupon is a percentage of the principal

The First Set of Cashflows is Debt

- Remember what that was

  "You get $6 every year for ten years,
  then a lump sum payment of
  $100 in ten years"

- The lump sum of $100 was the principal
- The $6 was the coupon payment
  - $6 = 6% of $100

Would You Pay $100

- Seems like a reasonable price
- Total of at least $160
  - You get a total of $60 (6 times 10) over 10 years
  - You also get your money back in ten years
- Problems
  - You don't get to use you $100 for ten years
  - Giving me $60 over ten years may not be enough

A Little Comparison Shopping

- Lets say you were going to pay B for the first set of cash flows
- But instead you are offered what's behind door number 2:
  - You can invest risk free at R% every year for the next ten years
- We'll call this the rolling strategy
  - You roll your investment as it matures every year

Now We'll Run a Horse Race: Who Wins After Ten Years?

- Rolling Strategy
  - Assume you invest B in this rolling strategy, and reinvest each year the entire amount

After Year 1: $\$B\ (1 + R)^1$

Year 2: $\$B\ (1 + R)^2$

Year 3: $\$B\ (1 + R)^3$

.

.

Year 10: $\$B\ (1 + R)^{10}$

Now, The First Set of Cashflows

- Assume you pay B and you reinvest the interim cashflows at R as well
- The first coupon is worth more in ten years due to reinvestment

Year 1 you get \$6

In year 2, after reinvesting at R, you get $6(1 + R)$

In year 3, you get $6(1 + R)(1+R) = 6(1 + R)^2$

.

.

By year 10, this grows to $6(1 + R)^9$

This Applies to All of The Coupons

Year 1 Cashflow In ten years: $6(1 + R)^9$
Year 2 Cashflow in ten years: $6(1 + R)^8$
Year 3 Cashflow in ten years: $6(1 + R)^7$
.
.
Year 10 Cashflow in ten years: 6 + 100

Total Cashflows in ten years,

$$100 + \sum_{i=1}^{10} 6\,(1 + R)^{10-i}$$

Now, Who Won the Race?

- It had better be a dead heat otherwise you have an arbitrage opportunity

Cashflow at the end of ten years of both strategies must be the same

Rolling Value in Ten Years = Cashflow One in Ten Years

$$B\,(1+R)^{10} = 100 + \sum_{i=1}^{10} 6\,(1+R)^{10-i}$$

What If They Are Different?

- You can retire in a few years and skip the rest of this course
- For example if

$$B(1+R)^{10} < 100 + \sum_{i=1}^{10} 6(1+R)^{10-i}$$

- Then borrow B dollars at R and buy the first security at B
  - No cost and you cash out in ten years
  - Do this a bunch of times and your making real money

So, We Figured Out They Must Be the Same

- This creates a simple pricing rule that we can use to answer our first question
- Just do a little division by $(1 + R)^{10}$ and you get the price of the first cashflow

$$\text{If } B(1+R)^{10} = 100 + \sum_{i=1}^{10} 6(1+R)^{10-i}$$

$$\mathbf{B} = \frac{6}{(1+R)^1} + \frac{6}{(1+R)^2} + \ldots + \frac{6}{(1+R)^N} + \frac{100}{(1+R)^N}$$

Of Course This Analysis Really Present Value And Future Value

- Future Value is the value at some future date of cash flows to be received in the future (or of funds invested today).

$$FV = PV\ (1 + R)^n$$

Where:

PV = Initial Investment

FV = Future value

R = Is the yearly investment rate rate

n = Number of years

Present Value

- Present Value is the value today of cash flows to be received in the future.

Present Value:

$$PV = \frac{FV}{(1+R)^n}$$

Where:

PV = Present value
FV = Future value
R = Reinvestment rate for future cash flows
n = Number of periods

Price, Reinvestment Rate, PV & FV

- The reinvestment rate links cashflows to price by present value and future value

$$B = \underbrace{\frac{6}{(1+R)^1}}_{} + \underbrace{\frac{6}{(1+R)^2}}_{} + \ldots + \underbrace{\frac{6}{(1+R)^{10}} + \frac{100}{(1+R)^{10}}}_{}$$

$$= \underbrace{\frac{FV_1}{(1+R)^1}}_{} + \underbrace{\frac{FV_2}{(1+R)^2}}_{} + \ldots + \underbrace{\frac{FV_{10}}{(1+R)^{10}}}_{}$$

$$= PV_1 + PV_2 + \ldots + PV_{10}$$

Yields And Reinvestment Rates

- The present value link to price and reinvestment rate is very powerful
- Given a reinvestment rate, R, you can find the price of a bond,B
  - The sum of the present values of the future cashflows
- Given a price, B, you can find the reinvestment rate, R
  - The reinvestment rate that sets the present value of the future cashflows equal to the price
- In the bond world, this reinvestment rate is called the **YIELD, Y**
  - So, Y = R

Equity and Debt vs Stocks and Bonds

- Stocks and bonds are securitized forms of equity and debt
- Securitization is just a categorization because we can still buy and sell debt in the form of loan
- But remember they are just cashflows

Yield

Zen and The Art of Bond Pricing

- In reality both the reinvestment rate and the yield are made up numbers
  - The one year rate that you can reinvest your cashflows at is not certain, but changes over time
- In reality just like for stocks, the market weighs in a lot of factors and comes up with a price for a debt cashflow
- The reinvestment rate is simply an attempt to back out a number that will discount the cashflows back to the market price

Yield To Maturity:

Yield is the reinvestment rate that equates the present value of the cash flows (interest and principal) to the market price.

$$\text{Price} = \frac{C}{(1+Y)^{1}} + \frac{C}{(1+Y)^{2}} + \ldots + \frac{P+C}{(1+Y)^{N}}$$

Where:

N = Years until maturity
C = Annual Coupon
P = Principal
Y = Yield to maturity

What Does Yield Measure?

- **Current Yield**
  - Takes only coupon interest into account

- **Yield to Maturity (YTM)**
  - Takes three components of return into account with some key assumptions:
    - Reinvest coupon income at the YTM
    - Hold to maturity
    - Cash flows are known and certain

How Do You Actually "Earn" the YTM?

The YTM, Y, is the discount rate that equates the present value of the expected cash flows to the market present value:

$$\mathbf{PV} = \frac{\mathbf{C}}{(\mathbf{1}+\mathbf{Y})^{\mathbf{1}}} + \frac{\mathbf{C}}{(\mathbf{1}+\mathbf{Y})^{\mathbf{2}}} + \ldots + \frac{\mathbf{P}+\mathbf{C}}{(\mathbf{1}+\mathbf{Y})^{\mathbf{N}}}$$

Equivalently, YTM is the reinvestment rate such that the FV at maturity is:

$$\mathbf{FV = C\,(1+Y)^{n-1} + C(1+Y)^{n-2} + \;.\;.\;.\; + (100 + C)}$$

resulting in a periodic *rate of return,* Y, equal to the YTM:

$$\mathbf{(1 + Y)^{n} = FV/PV}$$

The Reinvestment Rate/Yield

- Key to this analysis is the reinvestment rate
  - It tells us how we can convert current dollars to future dollars
  - It is like an exchange rate
- However, this number is really a artifact
- Though fictitious, it is very useful for comparisons

Yield Tells Us More Than Prices

- Higher coupon, higher price, but is it high enough? No, but does that mean the 10-3/4% is cheap
- Same coupon, longer maturity, lower price, why?--Yield curve inverted

			Yield To	
Coupon	Maturity	Price	Maturity	Call
5.500	02/28/03	98-092	6.236	
5.500	03/31/03	98-082	6.226	
5.750	04/30/03	98-266	6.219	
10.750	05/15/03	111-042	6.271	
5.500	05/31/03	98-062	6.210	

For Example

- Which ten year would you buy
  - One with a coupon of 6.5% price at $102-23 or
  - One with a coupon of 5.5% priced at $94-29?
- Yield levels out the playing field allowing us to compare apples with apples
- You would pick the 5.5% because it has the same risk and maturity but a higher yield
  - The 5.5% has a yield of 6.25%
  - The 6.5% has a yield of 6.2%

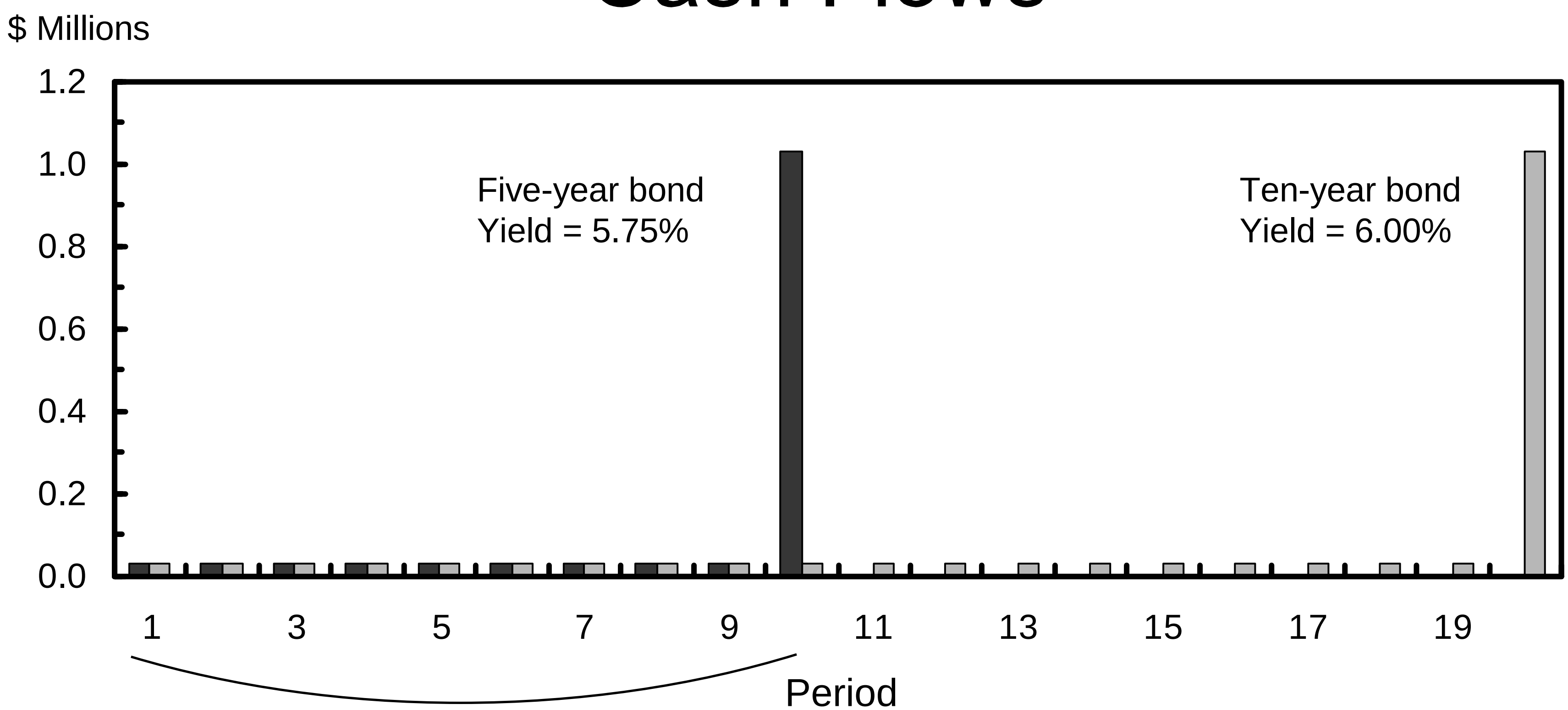
The Key To Understanding Bonds

- Yield goes up, price goes down
- Price goes up, yield goes down

Some problems with Yield as a Measure of Value

- How do you compare bonds with different maturities?
- How do you interpret the yield of a bond which has cash flows which vary as a function of interest rates?
- What about instances where the bond’s cash flows are not entirely certain for other reasons (e.g. risk of default)?

Consider Two Bonds, Both With Certain Cash Flows



Does it make sense to discount these cash flows at different rate?

Yield to Maturity: Uses and Drawbacks

***Definition*: the single discount rate that, when used to discount all of the cash flows on the bond over their appropriate periods to maturity, results in a present value equal to the bond's observed price in the market.**

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{100}{(1+r)^n}$$

***P* = observed price, *r* = yield to maturity, *C* = coupon payment, *n* = maturity**

Assumptions

- **All cash flows are discounted at the same rate**
- **All coupon income is re-invested at the same rate**
- **The bond is held to maturity**

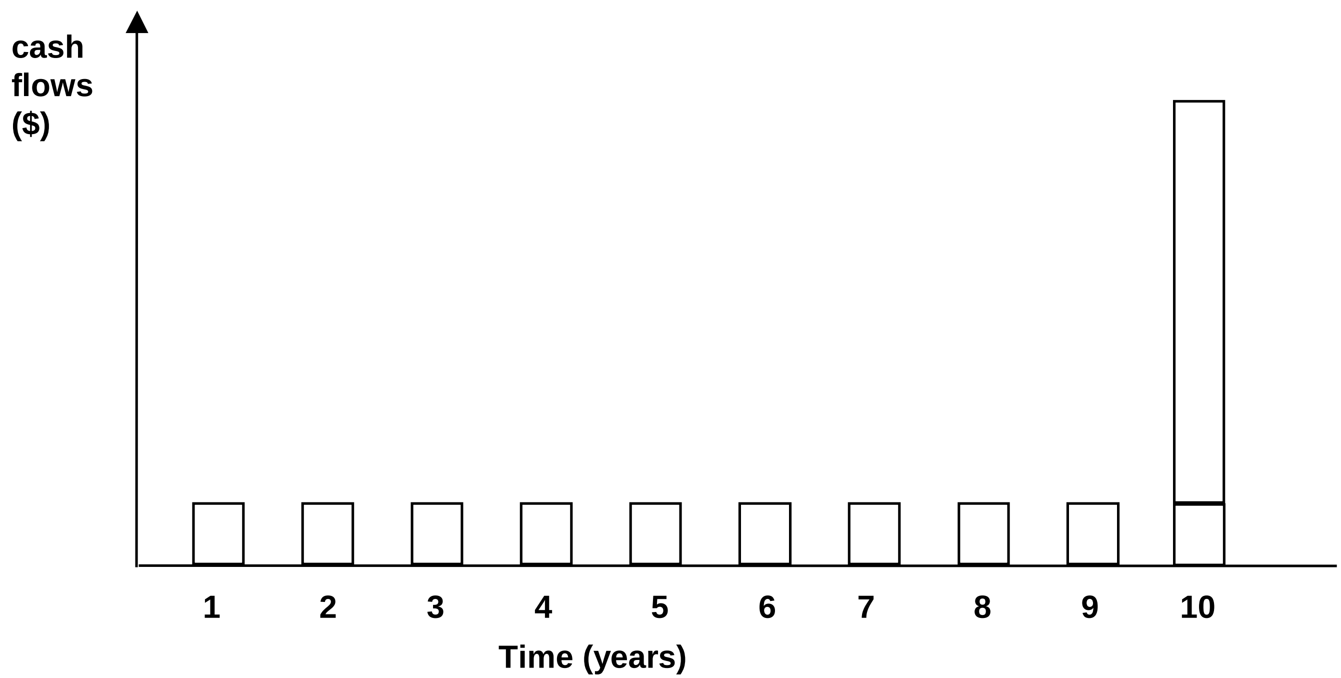
So Why Do We Use Yield At All?

- Market standard for discussing value in most markets -- including some where it shouldn’t be
- Easy to calculate -- anyone can do it without sophisticated analytics
- Analytically unambiguous -- not a “black box.” Everyone get the same results

Spot Curves

The Trick to Dealing With Different Maturities

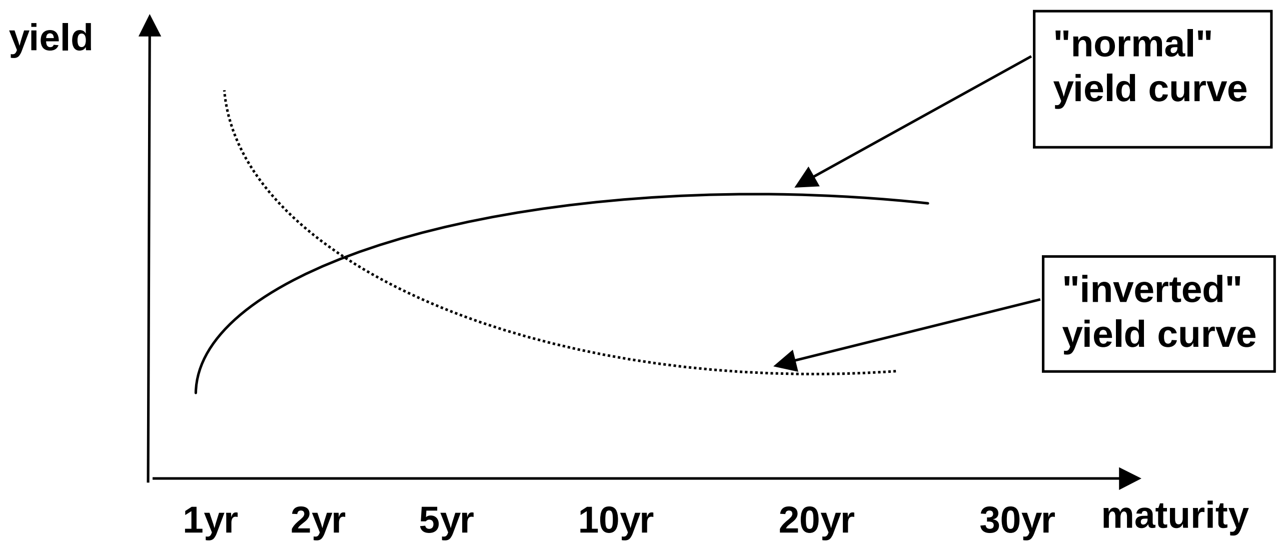
- **Think of bond cash flows as a series of zero-coupon bonds.**
- **Discount each cash flow at the appropriate discount rate based on on time to maturity.**



The Term Structure of Interest Rates Helps Us Figure This Out

Relationship between the yield and the maturity of bonds in the US Treasury market.

- **No default risk**
- **Most liquid bond market**



The Key To Using the Coupon Yield Curve is The Spot Rate

- The spot rate at a given maturity is the yield on the implied zero coupon bond with that maturity and, therefore, is the appropriate discount rate for valuing any cash flow occurring on that date

Spot Rate Analysis

- The spot curve is the fundamental building block for the valuation of any fixed income security
- A bond can be decomposed into a portfolio of zero coupon bonds
- The relationships among the spot rates depend on the shape of the yield curve.
- A fitted spot rate curve can be constructed from a universe of securities to provide a consistent representation of the term structure of interest rates

Calculating Spot Rates from the Coupon Curve: The Bootstrap Method (1)

Maturity (months)	Coupon (%)	Price	Yield (%)	Spot Rate (r)
6	4.00	100.32	3.340	?
12	4.50	100.78	3.700	?
18	3.75	99.72	3.940	?
24	4.00	99.70	4.160	?

Calculating Spot Rates from the Coupon Curve: The Bootstrap Method (2)

Maturity (months)	Coupon (%)	Price	Yield (%)	Spot Rate (r)
6	4.00	100.32	3.340	3.340
12	4.50	100.78	3.700	?
18	3.75	99.72	3.940	?

The 6-month maturity coupon bond has only one cash flow at maturity. It is equivalent to a zero coupon bond.

Calculating Spot Rates: The Bootstrap Method (3)

Maturity (months)	Coupon (%)	Price	Yield (%)	Spot Rate (r)
6	4.00	100.32	3.340	3.340
12	4.50	100.78	3.700	3.702
18	3.75	99.72	3.940	?

$$\text{Price} = \text{PV} = \frac{CF_6}{\left(1+\frac{r_6}{200}\right)} + \frac{CF_{12}}{\left(1+\frac{r_{12}}{200}\right)^2}$$

$$100.78 = \frac{2.25}{\left(1+\frac{3.340}{200}\right)} + \frac{102.25}{\left(1+\frac{r_{12}}{200}\right)^2}$$

$$r_{12} = 3.702$$

Calculating Spot Rates : The Bootstrap Method (4)

Maturity (months)	Coupon (%)	Price	Yield (%)	Spot Rate (r)
6	4.00	100.32	3.340	3.340
12	4.50	100.78	3.700	3.702
18	3.75	99.72	3.940	3.951

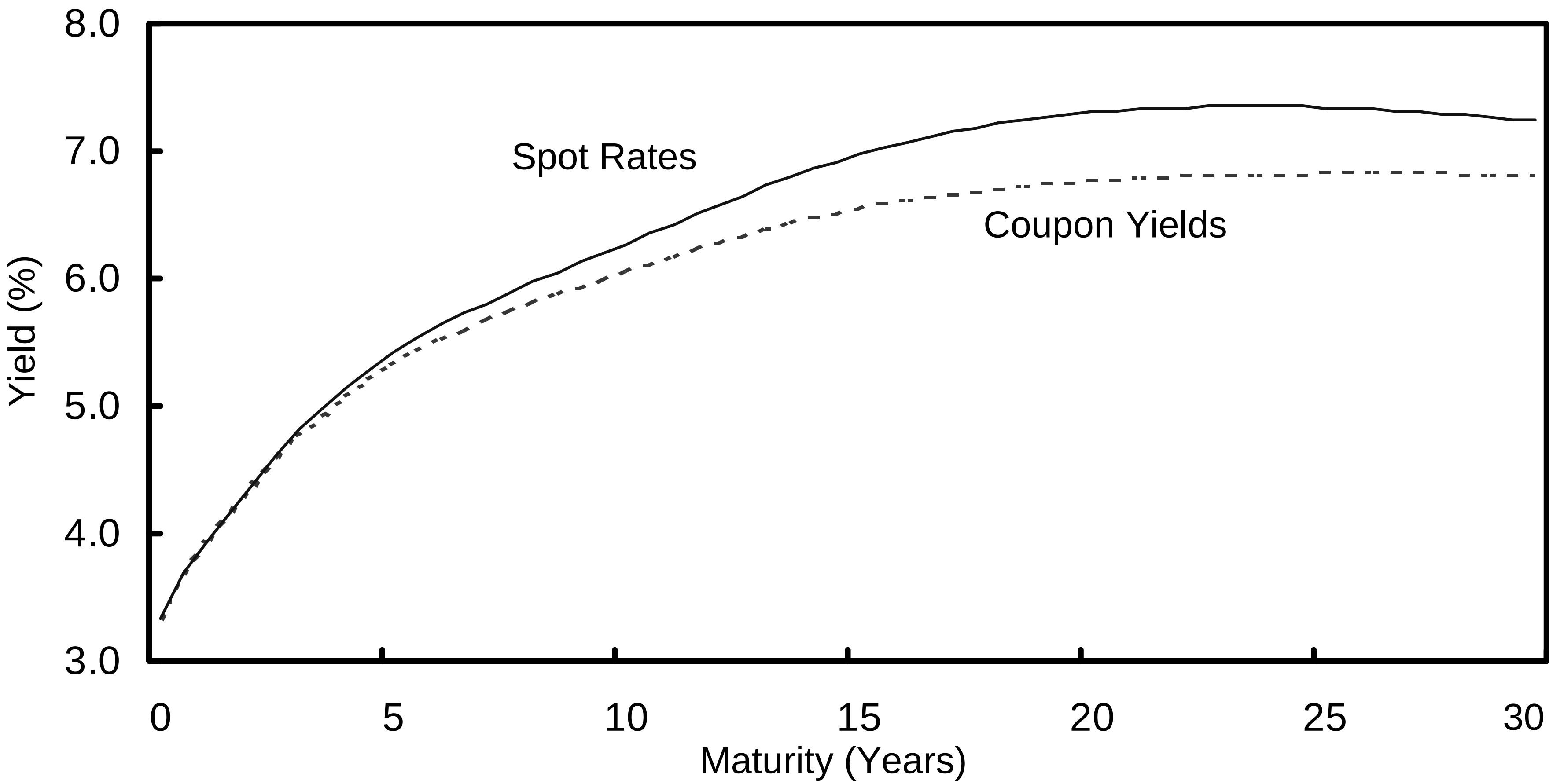
$$99.72 = \frac{1.875}{\left(1+\frac{3.340}{200}\right)} + \frac{1.875}{\left(1+\frac{3.702}{200}\right)^2} + \frac{101.875}{\left(1+\frac{r_{18}}{200}\right)^3}$$

$$r_{18} = 3.951$$

Calculating Spot Rates from the Coupon Curve: The Bootstrap Method (5)

Maturity (months)	Coupon (%)	Price	Yield (%)	Spot Rate (r)
6	4.00	100.32	3.340	3.340
12	4.50	100.78	3.700	3.702
18	3.75	99.72	3.940	3.951
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
120	6.00	100.00	6.000	6.200

The U.S. Treasury Coupon Yield Curve vs. the Spot Rate Curve



Valuing a Bond Using the Spot Curve

- What is the price of a 6% 10-year treasury?
- Each cash flow is discounted to its present value by the appropriate spot rates(s).

$$PV = \frac{CF_1}{\left(1+\frac{r_1}{200}\right)^1} = \frac{3}{\left(1+\frac{3.340}{200}\right)^1} = 2.951$$

$$PV_2 = \frac{CF_2}{\left(1+\frac{r_2}{200}\right)^2} = \frac{3}{\left(1+\frac{3.702}{200}\right)^2} = 2.892$$

$$PV_{20} = \frac{CF_{20}}{\left(1+\frac{r_{20}}{200}\right)^{20}} = \frac{103}{\left(1+\frac{6.200}{200}\right)^{20}} = 55.931$$

– Price of the bond is the sum of all its present values.

Valuing a Bond Using the Spot Curve (Continued)

Maturity (Yrs)	Cash Flow ($)	Spot Rate (%)	Present Value
0.5	3.0	3.340	2.951
1.0	3.0	3.702	2.892
1.5	3.0	3.951	2.829
...	...	...	...
10.0	103.0	6.200	55.931

Total Present Value = 100.000

Calculating the Discount Function

Maturity (months)	**Spot Rate (%)**	**Discount Function**
6	3.340	0.9836
12	3.702	0.9640
18	3.951	0.9430

$$PV_n = \frac{CF_n}{\left(1+\frac{r_n}{200}\right)^n}$$

$$\text{Discount Function} = DF_n = \frac{\$1}{\left(1+\frac{r_n}{200}\right)^n}$$

The discount function is the set of prices of implied zero coupon bonds at all future maturities, assuming each bond has a $1 par amount.

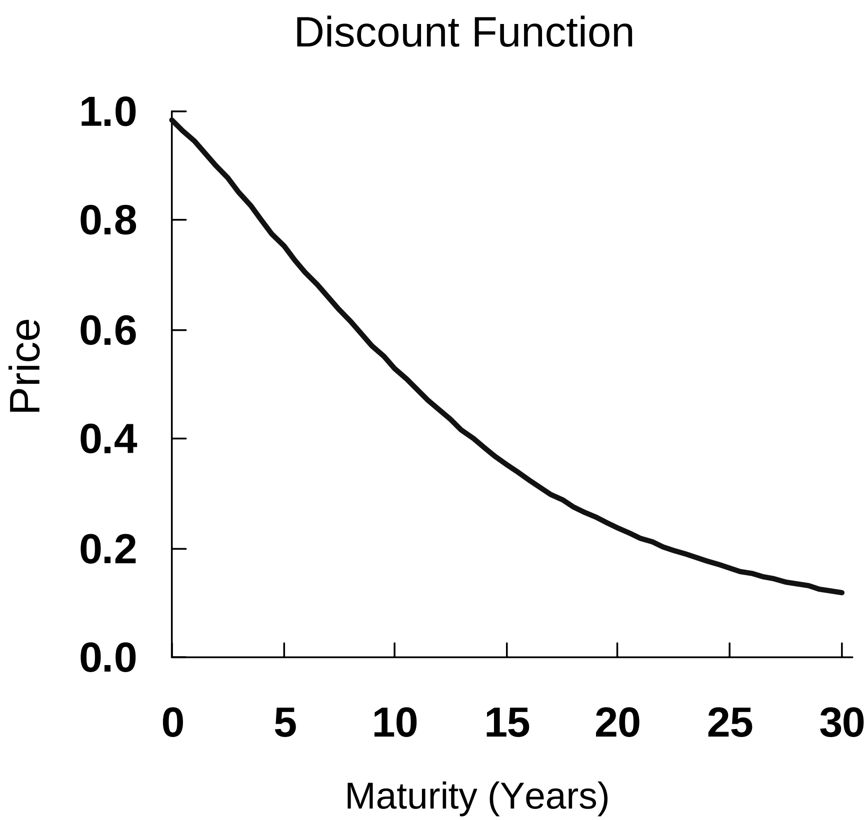
So, Price is the Sum of the Cashflows Scaled by the Discount Factors

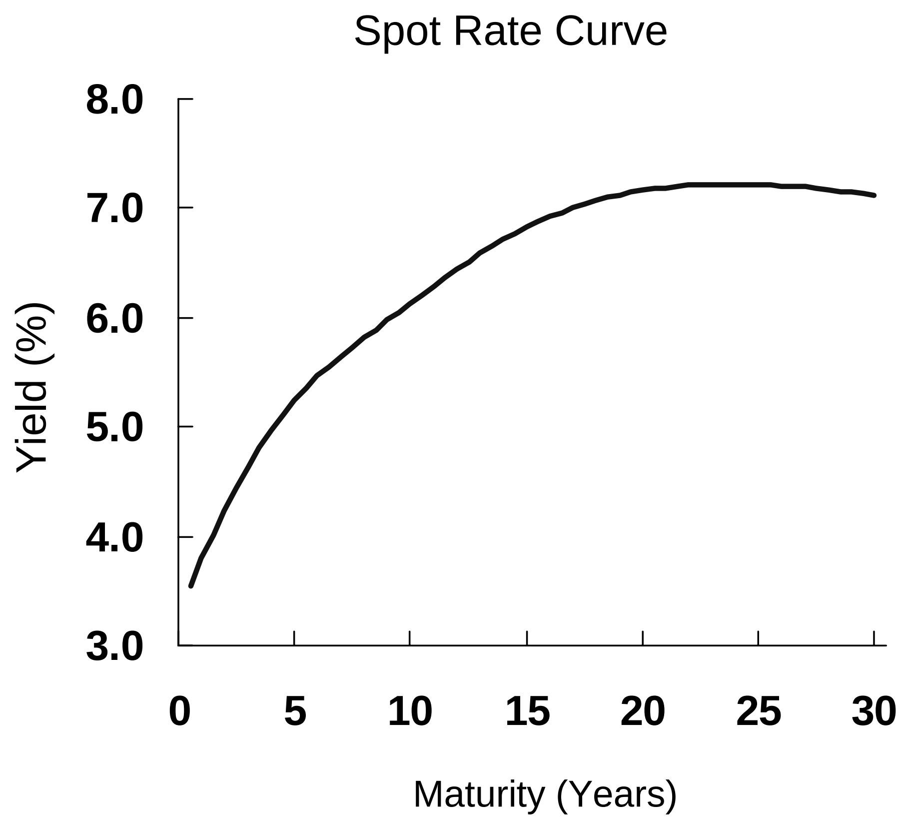
$$PV_n = \frac{CF_n}{\left(1+\frac{r_n}{200}\right)^n}$$

$$\text{Discount Function} = df_n = \frac{\$1}{\left(1+\frac{r_n}{200}\right)^n}$$

$$\text{Bond Price} = \sum_{i=1}^{n} df_i \, CF_i$$

The Spot Rate Curve and the Discount Function





Valuing a Bond Using the Discount Function

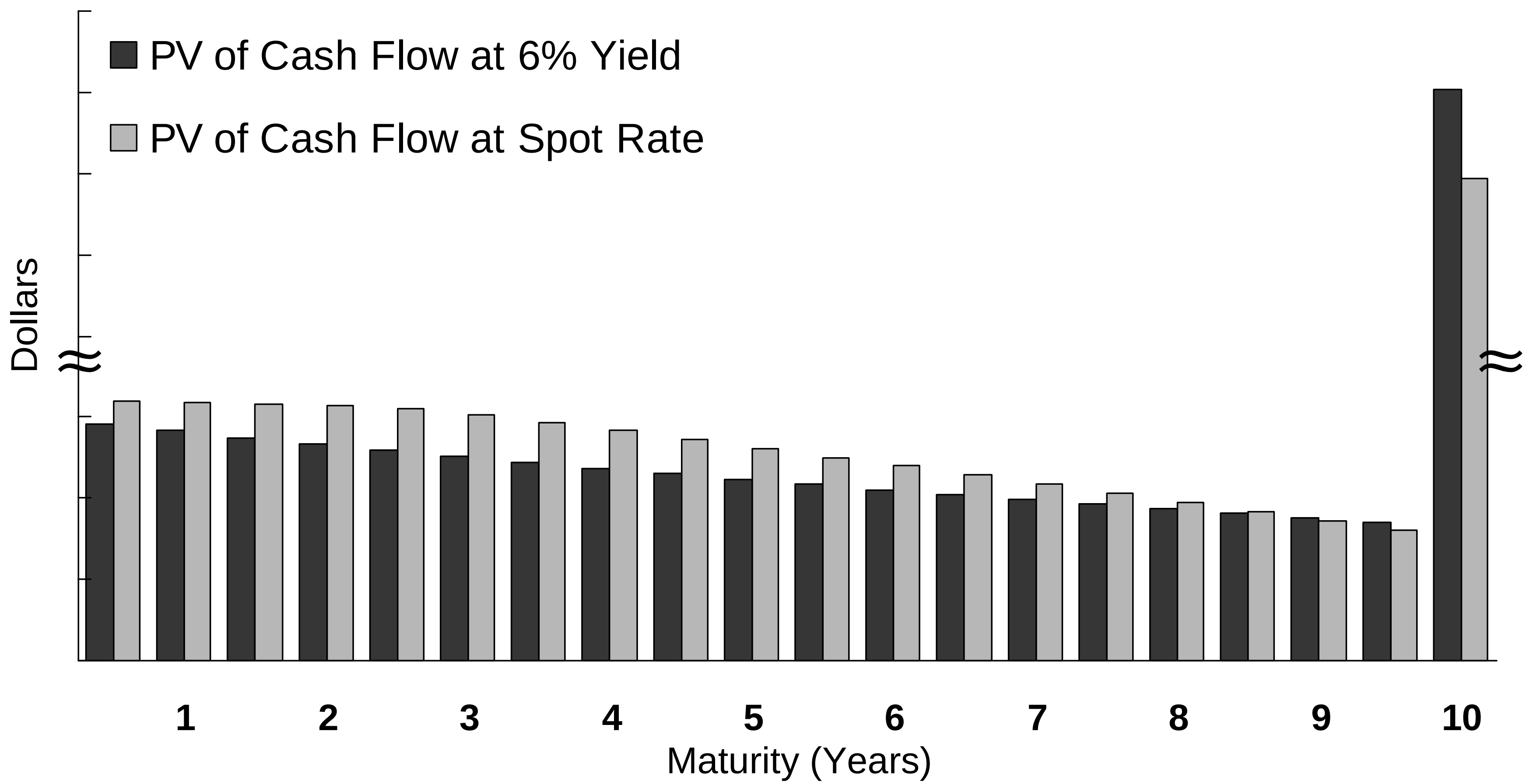
Maturity (Yrs)	Spot Rate (%)	Discount Function	x	Cash Flow ($)	=	Present Value
0.5	3.340	0.9836		3.0		2.9510
1.0	3.702	0.9640		3.0		2.8920
1.5	3.951	0.9430		3.0		2.8290
...	...	...		...		...
10.0	6.200	0.5430		103.0		55.9310

Total Present Value = 100.000

Not surprisingly, each present value is identical to its value using spot rate discounting.

Discounting Using Yield vs. Spot Rates

The Total Present Value is 100 in both cases.



Why are Treasury Prices Set by the Spot Curve?

- Yields are only an approximate pricing measure
- Potential arbitrage opportunities keep treasury prices in line
- Consider the case of a treasury priced below its spot curve-derived price
  - Arbitrage via stripping
- Consider the case of a treasury priced above its spot curve-derived price
  - Arbitrage via reconstitution
  - Opportunity to create a cheaper synthetic security or portfolio

Shortcomings of Yield and Spot Rates

- **The assumptions underlying Yield are often invalid:**
  - Coupon income might not be reinvested at the YTM: reinvestment risk
  - Bond might not be held to maturity (and YTM at horizon might not be the same as at present): price risk
  - Cash flows might not be certain or fixed (in amount and timing)
- **Total ROR scenario analysis can overcome some of these shortcomings**

What Features Would a Superior Measure Of Valuation Have?

- Address cash flows uncertainly -- magnitude or timing.
- Allow for terminal dates before maturity
- Allow reinvestment at user-specified rate that can be the same for different securities
- Allow user to analyze and combine different market scenarios

Total Rate of Return

Total Rate of Return

- What Is Total Rate of Return (TROR)?
- Computing TROR
- Sensitivity of TROR to Assumptions
- Expected Return

What Are the Components of Return from an Investment in Bonds?

- Coupon income
- Interest on coupon income
- Capital gains

The FV Can Be Decomposed According to the Three Components of Return

FV = horizon price + coupon income + interest on coupon income

where:

horizon price = R

coupon income = n x C/2

interest on coupon income

$$= \underbrace{[(C/2)(1+y/2)^{n-1} + (C/2)(1+y/2)^{n-2} + \ldots + C/2]}_{\text{FV of the coupon annuity}}$$

Total Rate of Return, Simply Put

- The easiest way to remember total return is:

$$\frac{100 * [(\text{Dollars at end}) - (\text{Dollars at beginning})]}{\text{Dollars at beginning}}$$

- Best not to annualize
  - Does it make sense to annualize a price return over a one-month period into an annualized return?

What You Need to Know...

- To calculate a total return, you need to know:
  - beginning price
  - ending price
  - holding period
  - reinvestment rate
  - interim cashflows
  - bond principal balance (factor) at horizon (if applicable)
  - compounding assumptions

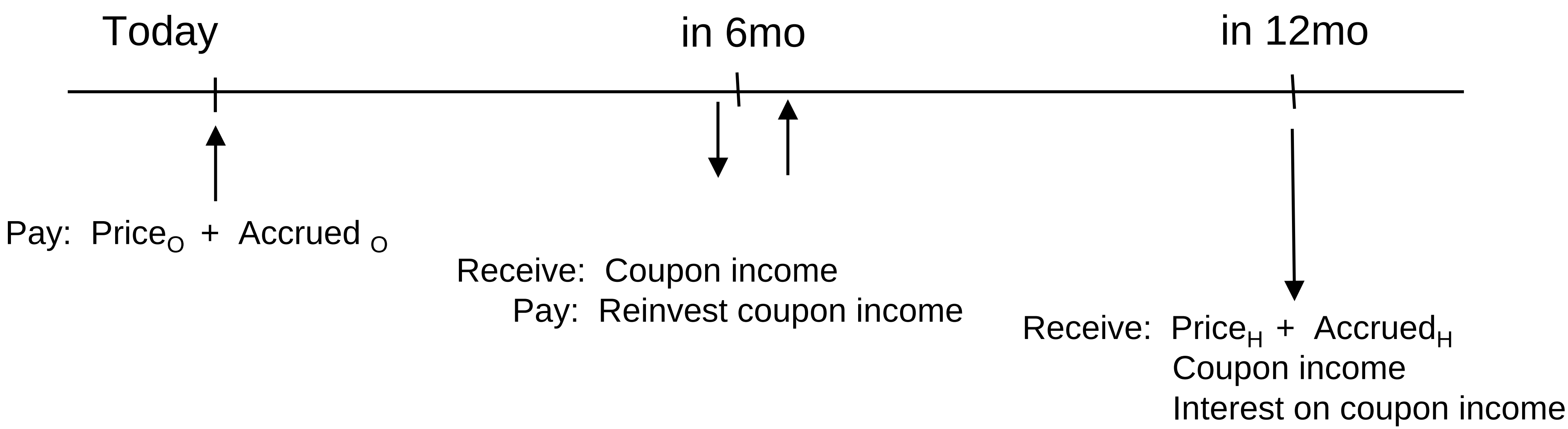
Calculating Total Rate of Return: An Example

- Consider buying $1MM of a 10-year Treasury today with a 6% coupon at $100-06 (5.975%) and selling it in a year (assuming cashflows occur on coupon dates).

I. Calculate the TROR assuming the yield on the bond will be 5.90% at horizon and the (bond equivalent) reinvestment rate will be 5.28%.

II. Calculate the TROR assuming the horizon yield on the bond will be 5.47% and the (bond equivalent) reinvestment rate will be 4.78%.

Calculating Total Rate of Return: The Cash Flows



Calculating Total Rate of Return: Case I

Now:	$Price_O + Accrued_O$	=	$1,001,875
	Total	=	$1,001,875
Horizon:	$Price_H + Accrued_H$	=	$1,006,906
	Coupon income = 2 x Coupon/2	=	$60,000
	Interest on 1st Coupon	=	$780
	Interest on 2nd Coupon	=	$0
	Total	=	$1,067,686
	$\frac{FV}{PV} - 1$	=	6.57%

Calculating Total Rate of Return: Case II

Now:	$Price_O + Accrued_O$	=	\$1,001,875
	Total	=	\$1,001,875
Horizon:	$Price_H + Accrued_H$	=	\$1,037,277
	Coupon income = 2 x Coupon/2	=	\$60,000
	Interest on 1st Coupon	=	\$717
	Interest on 2nd Coupon	=	\$0
	Total	=	\$1,097,994
	$\frac{FV}{PV} - 1$	=	9.59%

The Impact of Assumptions Varies as a Function of the Holding Period

- The longer the holding period:
  - The more influence the reinvestment rate has on the total rate of return
  - The less influence the horizon price has on the total rate of return

What Happens as the Holding Period Changes?

One-Year Holding Period,
$100-06 price; re-invest at yield

Bond	Yield	TROR	Change In TROR: Reinv Rate +100 bp	Change In TROR: Horizon Yld +25 bp
10-Year Treasury, 6% coupon	5.523	5.97	0.02	-1.66

Three-Year Holding Period,
$100-06 price; re-invest at yield

Bond	Yield	TROR	Change In TROR: Reinv Rate +100 bp	Change In TROR: Horizon Yld +25 bp
10-Year Treasury, 6% coupon	5.523	6.829	0.07	-0.41

What Is Scenario Analysis?

- The use of TROR to assess bond or portfolio performance over some investment horizon.
- It can be performed consistently on one or more securities of diverse types.
- When a variety of interest rate assumptions are applied, it can be an especially effective means of comparing the potential performance of alternative investments.
- It allows for adjustment to assumptions, including horizon pricing, reinvestment, prepayments, volatility, etc.

Why Does Total Rate of Return Fall Short as a Comprehensive Measure of Value?

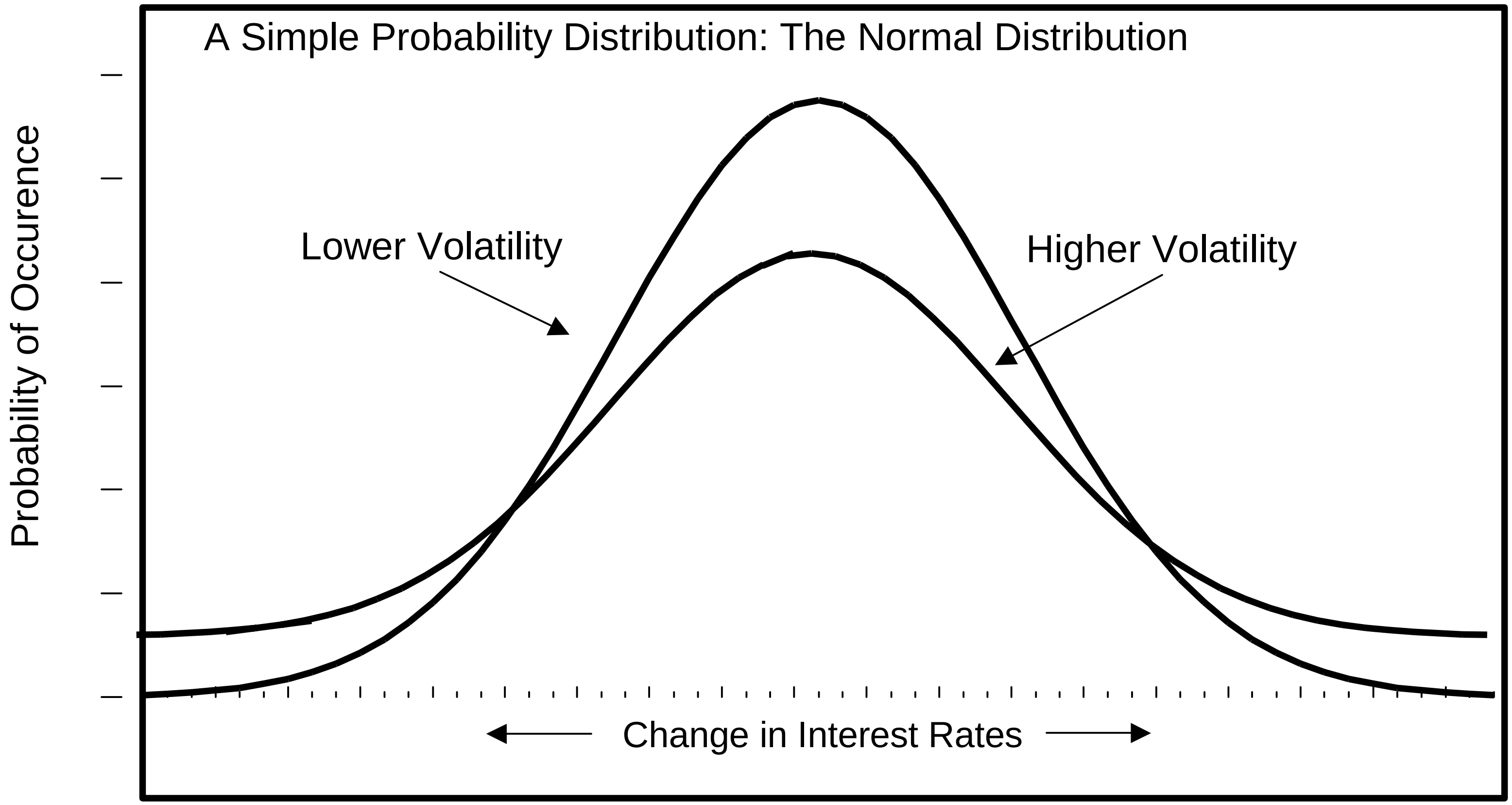
Consider the following bond swap:
(1 year horizon)

Scenario Rates of Return
Yield Changes (bp)

	Bond	Price	Yield	-200	-100	0	100	200
Buy	Cpn STRIP 11/15/24	22.05	5.83	63.52	32.82	5.83	-17.93	-38.85
Sell	UST 6.500 5/15/05	105-15	5.52	15.06	10.21	5.52	1.00	-3.36
	Difference		**0.31**	**48.46**	**22.61**	**0.31**	**-18.93**	**-35.49**

Which rate of return scenario best captures the relative value?

The Probability of Different Interest Rate Environments



Calculating Expected Return

- Expected return is calculated by weighting different scenario results by the probabilities of those scenarios
- The choice of scenario probabilities could be based upon:
  - Historical probabilities
  - The market's pricing of probabilities in the options market

Consider Our Bond Swap...

Scenario Rates of Return
Yield Changes (bp)

	Bond	Price	Yield	-200	-100	0	100	200	Expected Return
Buy	Cpn STRIP 11/15/24	22.05	5.83	63.52	32.82	5.83	-17.93	-38.85	8.13
Sell	UST 6.500 5/15/05	105–15	5.52	15.06	10.21	5.52	1.00	-3.36	5.64
	Difference		0.31	48.46	22.61	0.31	-18.93	-35.49	2.49
	Scenario Probability			12%	23%	30%	23%	12%	

Summary of TROR (Scenario) Analysis

- In contrast to YTM, it allows you to generally and explicitly specify the assumptions for all sources of bond return
- The relative sensitivity of TROR to reinvestment rates and horizon yield varies as a function of holding period
- Allowing for uncertainty in these assumptions leads to a better measure of relative value: expected return

Gee, This Is Easy

- The labels of debt and equity are just that, labels
- The same risk means, the same return
- Look at a bond as a bundle of cashflows
- Total returns is a better measure of value because it takes into account
  - Mark-to-market
  - Reinvestment risk

And Remember

Yield goes up, price goes down

Appendix

The Effect of Compounding

- Compounding adds value -- it increases the present value of a bond.
- For instance, a semi-annual pay bond is worth more than a bond that pays the same interest rate annually.
- Why? Because early cash flows are worth more than late cash flows in present value terms.
- You can earn interest on your interest.

Accounting for Compounding

- The Present Value formula can easily account for different payment periods:

$$PV = \frac{FV}{(1+R)^N}$$

Where:

m = Payments per year
n = Number of years
N = Number of interest payments (m × n)
R = Periodic interest payment (r/m)

There are Many Different Kinds of Yields

- Bond-equivalent (or semi-annual) yield (“BEY”)
- Annual yield
- Money market yield
- Yield to Call
- “Option-adjusted” yield
- Yield to Worst
- Current yield
- And others!

Yield to Maturity: Other Types

For an annual yield:

$$\text{Price} = \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \ldots + \frac{R+C}{(1+y)}$$

For money market yield:

$$\text{Price} = \frac{100}{1+\frac{yD}{360}}$$

For discount yield (T-Bills):

$$\text{Price} = 100 \left(1 - \frac{yD}{360}\right)$$

Where D = days to maturity

Day Count: 30/360 Calendar

- To determine the number of 30/360 days between two dates, $Date_1$ and $Date_2$, where $Date_1$ is earlier :

$$\begin{aligned}
&(360\,(Year_2 - Year_1) \\
&+ 30\,(Month_2 - Month_1) \\
&+ \text{If } Day_1 < 30, Day_2 - Day_1 \\
&+ \text{If } Day_1 = 30 \text{ or } 31, (\text{the smaller of } Day_2 \text{ and } 30) - 30) \div \\
&\ 30/360 \text{ days between } Date_1 \text{ and } Date_2
\end{aligned}$$

- When either date falls on February 28 or 29 the conventions vary.

Accrued Interest

- The amount of interest due but unpaid between the last coupon date and the settlement date.

  Examples:

  Fixed rate, semi-annual

$$\frac{C}{2} \bullet \frac{\#\text{ of days since last coupon}}{\#\text{ of days from last coupon to next coupon}}$$

  Floating rate, actual/360

$$C \bullet \frac{\#\text{ of days since last coupon}}{360}$$

- Present value = Price + Accrued Interest

Calendars

- **Example:** With a settlement date of August 1, 1998 and a Actual/Actual day calendar, the accrued interest on the 8% due November 15, 2021 UST would be:

$$4\% \bullet \frac{\text{Act/Act Days Between May 15, 1998 and August 1, 1998}}{\text{Act/Act Days Between May 15, 1998 and November 15, 1998}} = 4\% \bullet \frac{78}{184} = 1.696\%$$

- **Example 2:** With a settlement date of September 30, 1998 and a “30/360” day calendar, the accrued interest on a 6% corporate due January 31, 2007 would be:

$$3\% \bullet \frac{\text{30/360 Days Between July 31, 1998 and September 30, 1998}}{\text{30/360 Days Between July 31, 1998 and January 31, 1999}} = 3\% \bullet \frac{60}{180} = 1.000\%$$

One Bond's Yield Calculated Four Ways

Wal-Mart corporation has an outstanding 8.5% bond that matures in 25 years and is callable at 104-08 on or after 6 years from now and callable at par on or after 16 years from now. Its recent price is $114.50

• Current Yield:	7.42%
• Bond-equivalent yield:	7.24%
• Yield to next call:	6.17%
• Yield to par call	6.98%