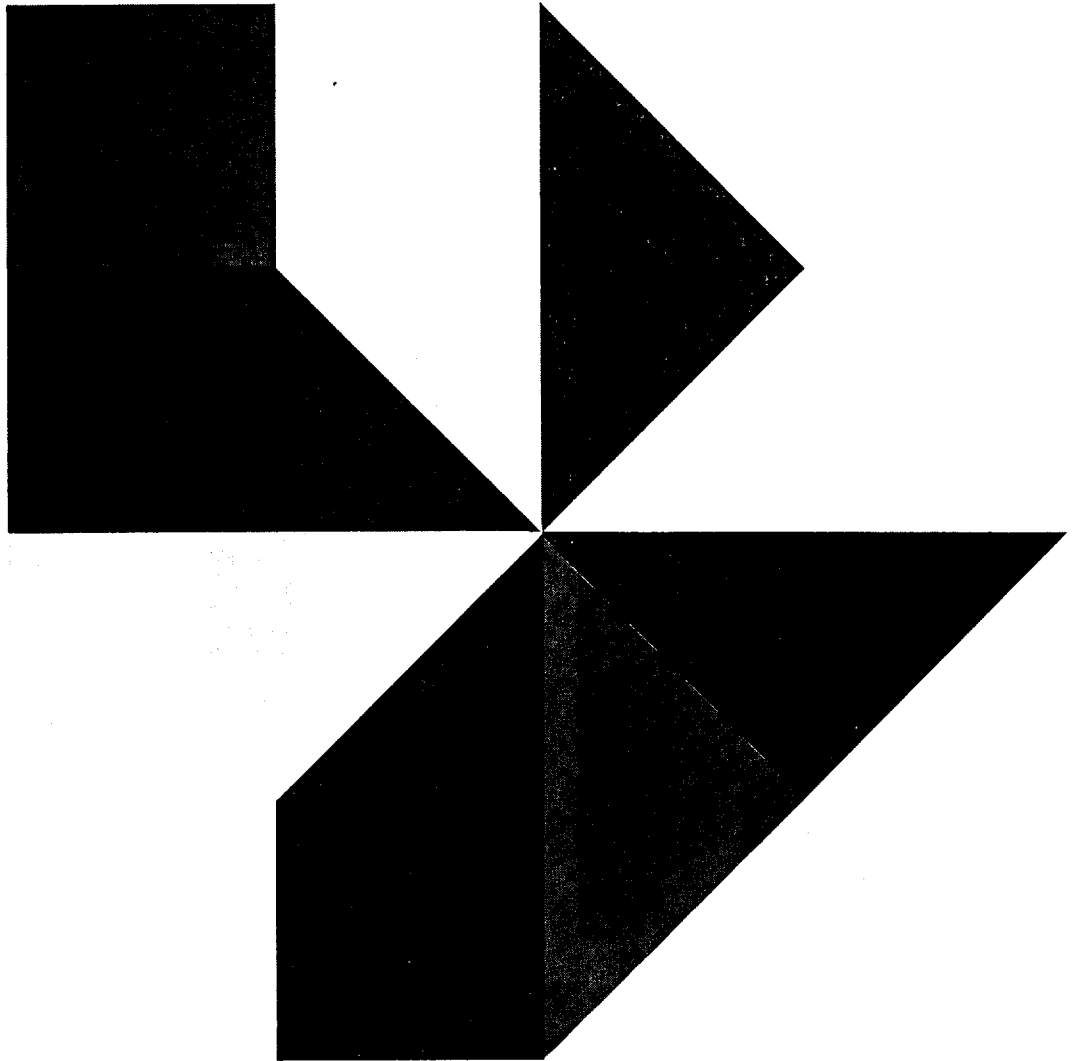


Introduction to Finite Mathematics

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Finite Mathematics

3rd edition

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Third Edition

John G. Kemeny / J. Laurie Snell /^{*} Gerald L. Thompson

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Preface to the Third Edition

The term *Finite Mathematics* was first used in the title of the first edition of this book. Since that time it has been generally accepted to describe those topics in modern mathematics that do not depend upon limiting processes, derivatives, or other infinite concepts and that have important real-world applications.

The purpose of the first edition of this book was to introduce college students to the elementary theory of logic, sets, probability theory, and linear algebra and to treat a number of practical applications either from everyday situations or from applications to the biological and social sciences. This central idea has been retained in the third edition of the book; however, experience has shown the desirability of adding additional topics. We have therefore treated the original topics more concisely and added some new subjects and new treatments of old subjects.

The core material of the book consists of the first four chapters. Chapter 1 is a brief introduction to the elementary logic of statements. Chapter 2 contains the basic ideas of the theory of sets and also introduces some fundamental counting techniques. These two chapters constitute a condensed version of the first three chapters of the earlier editions. They contain all the material necessary for the later topics, but some of the more esoteric topics have been eliminated. Chapter 3 is an introduction to finite probability theory and Chapter 4 introduces vectors and matrices and the solution of simultaneous equations. This core material constitutes a self-contained unit which may be used as an introduction to finite mathematics. Or, it may be supplemented in a wide variety of ways by selecting topics from the later chapters. We will discuss several such options presently.

The use of computers was in its infancy when the first edition was published and wide-scale use of time-shared computers for educational purposes became a reality just after the publication of the second edition.

Since *Finite Mathematics* lends itself ideally to computer treatment, and since computers can make the study of *Finite Mathematics* more interesting and more meaningful, we have included an introduction to computer programming in Chapter 5. We have chosen for this purpose the general-purpose computer language BASIC which is widely used in time-sharing systems. The advantage of this language is that the student can start writing computer programs very quickly, and yet it is flexible enough to allow the writing of the most complex computer program. Chapter 5 has been so organized that much of the material and many of the exercises may be taken up even if computers are not available to the students. However, the full impact of the chapter cannot be realized without giving "hands-on" experience for students. Such experience in writing and debugging their own computer programs both provides students with greater mathematical power and helps to reinforce the understanding of fundamental concepts.

One of the shortcomings of the earlier editions of *Finite Mathematics* was the fact that while they contained a good introduction to probability theory, they included little or nothing about the applications of that theory to statistics. For this reason we have added, in Chapter 6, an introduction to finite statistics. This is a natural outgrowth of the core material and leads to a further wealth of practical applications.

Since the appearance of the first edition, linear programming and matrix game theory have received widespread use for a wide variety of applications. We have therefore included, in Chapter 7, a completely revised and expanded treatment of these two important topics. The key technique used in solving large-scale problems in these two areas is the simplex method, and the treatment included in this book is due to A. W. Tucker.

Chapter 8 is devoted entirely to applications. We have retained several of these from earlier editions, and have added three new topics. The first is an application to two linear economic models which depend on the same underlying mathematical model. The second is an application of linear programming to a governmental decision problem. The third discusses the branch-and-bound method for the solution of two combinatorial decision problems.

The problems at the end of each section of the core chapters, which have been widely used (and widely copied!), have been completely revised. As has been our custom, we have tried to give two of each kind of exercise wherever possible, one with and one without an answer printed in the text. We hope that this fresh problem material will come as welcome change for repeat users of the book. We have also updated a number of problems dealing with topics that have become irrelevant since 1957, when the first edition was published.

We believe that this book can be used in many different ways. The basic core material of the first four chapters (even with the omission of the asterisked sections) constitutes a self-contained unit. A course in finite mathematics with an introduction to computing is contained in Chapters 1-5. (The computer material may also be integrated with Chapters 2, 3, and 4

rather than waiting until the first four chapters are completed.) A course in finite mathematics with an introduction to statistics can be designed by covering Chapters 1-4 and the first five sections of Chapter 6. For a more technical introduction to statistics, including an introduction to computing, the first six chapters form a natural unit. An introduction to finite mathematics and linear programming and games is contained in Chapters 1-4 and Sections 1-4 and Sections 3 and 8-10 of Chapter 7. For a more technical introduction of the same topics, one needs an introduction to computation and therefore all of Chapters 1-5 and 7 should be included. Covering the book in its entirety gives a good introduction to the mathematics used in behavioral and social sciences; the same goal may be achieved somewhat more briefly by judicious selection of topics from the last four chapters.

We wish to thank our colleagues in many institutions who have read the material and made comments and suggestions. Professor Frank Deane has been especially helpful in this respect. We are particularly grateful to Professor A. W. Tucker for showing a strong and continuing interest in our work and for suggesting a new approach to the simplex method. We thank Messrs. Mike Vitale and Ross Kindermann for supplying most of the new problem material. We thank Mrs. Bonnie Clark for her assistance in the preparation of the manuscript. We also thank Mrs. Eleanor Balocik for her work in preparing the Solutions Manual. To Dartmouth College and Carnegie-Mellon University we offer our appreciation for providing facilities (including computer usage) which made the preparation of this book possible. And finally we thank the staff of Prentice-Hall for their careful attention to editorial details.

J.G.K., J.L.S., G.L.T.