## SYMBOL DEFINITION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Acceleration (Sec. 10.2)</td>
</tr>
<tr>
<td>$a$</td>
<td>Horizontal distance from $W$ to toe of footing</td>
</tr>
<tr>
<td>$a_{max}$</td>
<td>Maximum horizontal acceleration at ground surface (also known as peak ground acceleration)</td>
</tr>
<tr>
<td>$A_P$</td>
<td>Anchor pull force (sheet pile wall)</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion based on total stress analysis</td>
</tr>
<tr>
<td>$c'$</td>
<td>Cohesion based on effective stress analysis</td>
</tr>
<tr>
<td>$c_u$</td>
<td>Adhesion between bottom of footing and underlying soil</td>
</tr>
<tr>
<td>$d$</td>
<td>Resultant location of retaining wall forces (Sec. 10.1.1)</td>
</tr>
<tr>
<td>$d_1$</td>
<td>Depth from ground surface to groundwater table</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Depth from groundwater table to bottom of sheet pile wall</td>
</tr>
<tr>
<td>$D$</td>
<td>Depth of retaining wall footing</td>
</tr>
<tr>
<td>$D'$</td>
<td>Portion of sheet pile wall anchored in soil (Fig. 10.9)</td>
</tr>
<tr>
<td>$e$</td>
<td>Lateral distance from $P_v$ to toe of retaining wall</td>
</tr>
<tr>
<td>$F$, $FS$</td>
<td>Factor of safety</td>
</tr>
<tr>
<td>$F_{SL}$</td>
<td>Factor of safety against liquefaction</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of retaining wall</td>
</tr>
<tr>
<td>$H'$</td>
<td>Unsupported face of sheet pile wall (Fig. 10.9)</td>
</tr>
<tr>
<td>$k_A$</td>
<td>Active earth pressure coefficient</td>
</tr>
<tr>
<td>$k_{AE}$</td>
<td>Combined active plus earthquake coefficient of pressure (Mononobe-Okabe equation)</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Seismic coefficient, also known as pseudostatic coefficient</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Coefficient of earth pressure at rest</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Passive earth pressure coefficient</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Vertical pseudostatic coefficient</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of active wedge at top of retaining wall</td>
</tr>
<tr>
<td>$m$</td>
<td>Total mass of active wedge</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>Maximum moment in sheet pile wall</td>
</tr>
<tr>
<td>$N'$</td>
<td>Sum of wall weights $W$ plus, if applicable, $P_v$</td>
</tr>
<tr>
<td>$P_A$</td>
<td>Active earth pressure resultant force</td>
</tr>
<tr>
<td>$P_H$</td>
<td>Pseudostatic horizontal force acting on retaining wall</td>
</tr>
<tr>
<td>$P_{HR}$</td>
<td>Pseudostatic horizontal force acting on restrained retaining wall</td>
</tr>
<tr>
<td>$P_F$</td>
<td>Sum of sliding resistance forces (Fig. 10.2)</td>
</tr>
<tr>
<td>$P_H$</td>
<td>Horizontal component of active earth pressure resultant force</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Lateral force due to liquefied soil</td>
</tr>
<tr>
<td>$P_P$</td>
<td>Passive resultant force</td>
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10.1 INTRODUCTION

A retaining wall is defined as a structure whose primary purpose is to provide lateral support for soil or rock. In some cases, the retaining wall may also support vertical loads. Examples include basement walls and certain types of bridge abutments. The most common types of retaining walls are shown in Fig. 10.1 and include gravity walls, cantilevered walls, counterfort walls, and crib walls. Table 10.1 lists and describes various types of retaining walls and backfill conditions.

10.1.1 Retaining Wall Analyses for Static Conditions

Figure 10.2 shows various types of retaining walls and the soil pressures acting on the walls for static (i.e., nonearthquake) conditions. There are three types of soil pressures acting on a retaining wall: (1) active earth pressure, which is exerted on the backside of the wall; (2) passive earth pressure, which acts on the front of the retaining wall footing; and (3) bearing pressure, which acts on the bottom of the retaining wall footing. These three pressures are individually discussed below.

Active Earth Pressure. To calculate the active earth pressure resultant force $P_a$, in kilonewtons per linear meter of wall or pounds per linear foot of wall, the following equation is used for granular backfill:

$$P_a = \frac{1}{2} k_a \gamma H^2$$

where $k_a$ = active earth pressure coefficient, $\gamma$ = total unit weight of the granular backfill, and $H$ = height over which the active earth pressure acts, as defined in Fig. 10.2. In its simplest form, the active earth pressure coefficient $k_a$ is equal to

$$k_a = \tan^2 (45^\circ - \frac{1}{2} \phi)$$
where $\phi =$ friction angle of the granular backfill. Equation (10.2) is known as the active Rankine state, after the British engineer Rankine who in 1857 obtained this relationship. Equation (10.2) is only valid for the simple case of a retaining wall that has a vertical rear face, no friction between the rear wall face and backfill soil, and the backfill ground surface is horizontal. For retaining walls that do not meet these requirements, the active earth pressure

**FIGURE 10.1** Common types of retaining walls. (a) Gravity walls of stone, brick, or plain concrete. Weight provides overturning and sliding stability. (b) Cantilevered wall. (c) Counterfort, or buttressed wall. If backfill covers counterforts, the wall is termed a counterfort. (d) Crib wall. (e) Semigavity wall (often steel reinforcement is used). (f) Bridge abutment. *(Reproduced from Bowles 1982 with permission of McGraw-Hill, Inc.)*
Table 10.1 Types of Retaining Walls and Backfill Conditions

<table>
<thead>
<tr>
<th>Topic</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Types of retaining walls</td>
<td>As shown in Fig. 10.1, some of the more common types of retaining walls are gravity walls, counterfort walls, cantilevered walls, and crib walls (Cernica 1995a). Gravity retaining walls are routinely built of plain concrete or stone, and the wall depends primarily on its massive weight to resist failure from overturning and sliding. Counterfort walls consist of a footing, a wall stem, and intermittent vertical ribs (called counterforts) which tie the footing and wall stem together. Crib walls consist of interlocking concrete members that form cells which are then filled with compacted soil. Although mechanically stabilized earth retaining walls have become more popular in the past decade, cantilever retaining walls are still probably the most common type of retaining structure. There are many different types of cantilevered walls, with the common feature being a footing that supports the vertical wall stem. Typical cantilevered walls are T-shaped, L-shaped, or reverse L-shaped (Cernica 1995a).</td>
</tr>
<tr>
<td>Backfill material</td>
<td>Clean granular material (no silt or clay) is the standard recommendation for backfill material. There are several reasons for this recommendation: 1. Predictable behavior: Import granular backfill generally has a more predictable behavior in terms of earth pressure exerted on the wall. Also, expansive soil-related forces will not be generated by clean granular soil. 2. Drainage system: To prevent the buildup of hydrostatic water pressure on the retaining wall, a drainage system is often constructed at the heel of the wall. The drainage system will be more effective if highly permeable soil, such as clean granular soil, is used as backfill. 3. Frost action: In cold climates, frost action has caused many retaining walls to move so much that they have become unusable. If freezing temperatures prevail, the backfill soil can be susceptible to frost action, where ice lenses form parallel to the wall and cause horizontal movements of up to 0.6 to 0.9 m (2 to 3 ft) in a single season (Sowers and Sowers 1970). Backfill soil consisting of clean granular soil and the installation of a drainage system at the heel of the wall will help to protect the wall from frost action.</td>
</tr>
<tr>
<td>Plane strain condition</td>
<td>Movement of retaining walls (i.e., active condition) involves the shear failure of the wall backfill, and the analysis will naturally include the shear strength of the backfill soil. Similar to the analysis of strip footings and slope stability, for most field situations involving retaining structures, the backfill soil is in a plane strain condition (i.e., the soil is confined along the long axis of the wall). As previously mentioned, the friction angle ( \delta ) is about 10 percent higher in the plane strain condition compared to the friction angle ( \phi ) measured in the triaxial apparatus. In practice, plane strain shear strength tests are not performed, which often results in an additional factor of safety for retaining wall analyses.</td>
</tr>
</tbody>
</table>

Coefficient \( k_A \) for Eq. (10.1) is often determined by using the Coulomb equation (see Fig. 10.3). Often the wall friction is neglected \( (\delta = 0^\circ) \), but if it is included in the analysis, typical values are \( \delta = \frac{1}{3} \phi \) for the wall friction between granular soil and wood or concrete walls and \( \delta = 20^\circ \) for the wall friction between granular soil and steel walls such as sheet pile walls. Note in Fig. 10.3 that when the wall friction angle \( \delta \) is used in the analysis, the active
earth pressure resultant force $P_A$ is inclined at an angle equal to $\delta$. Additional important details concerning the active earth pressure follow.

1. **Sufficient movement:** There must be sufficient movement of the retaining wall in order to develop the active earth pressure of the backfill. For dense granular soil, the amount of wall translation to reach the active earth pressure state is usually very small (i.e., to reach active state, wall translation $\leq 0.0005H$, where $H =$ height of wall).

2. **Triangular distribution:** As shown in Figs. 10.2 and 10.3, the active earth pressure is a triangular distribution, and thus the active earth pressure resultant force $P_A$ is located at a distance equal to $1/3H$ above the base of the wall.

3. **Surcharge pressure:** If there is a uniform surcharge pressure $Q$ acting upon the entire ground surface behind the wall, then an additional horizontal pressure is exerted upon the retaining wall equal to the product of $k_A$ and $Q$. Thus the resultant force $P_2$, in kilonewtons per linear
LOCATION OF RESULTANT

MOMENTS ABOUT TOE:
\[ d = \frac{Wa + P_\text{ve} - P_{Hb}}{W + P_v} \]
ASSUMING \( P_P = 0 \)

OVERTURNING

MOMENTS ABOUT TOE:
\[ F = \frac{Wa}{P_{Hb} - P_{ve}} \geq 1.5 \]

IGNORE OVERTURNING IF \( R \) IS WITHIN MIDDLE THIRD (SOIL), MIDDLE HALF (ROCK).
CHECK \( R \) AT DIFFERENT HORIZONTAL PLANES FOR GRAVITY WALLS.

RESISTANCE AGAINST SLIDING
\[ F = \frac{(W + P_v) \tan \delta + C_a B}{P_H} \geq 1.5 \]
\[ F = \frac{(W+P_v) \tan \delta + C_a B + P_P}{P_H} \geq 2.0 \]
\[ P_F = (W + P_v) \tan \delta + C_a B \]
\( C_a = \) ADHESION BETWEEN SOIL AND BASE
\( \tan \delta = \) FRICTION FACTOR BETWEEN SOIL AND BASE
\( W = \) INCLUDES WEIGHT OF WALL AND SOIL IN FRONT FOR GRAVITY AND SEMIGRAVITY WALLS.
INCLUDES WEIGHT OF WALL AND SOIL ABOVE FOOTING, FOR CANTILEVER AND COUNTERFORT WALLS.

FIGURE 10.2c  Design analysis for retaining walls shown in Fig. 10.2a and b. (Reproduced from NAVFAC DM-7.2, 1982.)
meter of wall or pounds per linear foot of wall, acting on the retaining wall due to the surcharge $Q$ is equal to $P_2 = \frac{QH}{k_A}$, where $Q$ = uniform vertical surcharge acting upon the entire ground surface behind the retaining wall, $k_A$ = active earth pressure coefficient [Eq. (10.2) or Fig. 10.3], and $H$ = height of the retaining wall. Because this pressure acting upon the retaining wall is uniform, the resultant force $P_2$ is located at midheight of the retaining wall.

4. Active wedge: The active wedge is defined as that zone of soil involved in the development of the active earth pressures upon the wall. This active wedge must move laterally to develop the active earth pressures. It is important that building footings or other

A) Coulomb’s Equation (Static Condition):

$$K_A = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \frac{\sin(\delta + \phi)\sin(\phi - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)} \right]^2}$$

B) Mononobe-Okabe Equation (Earthquake Condition):

$$K_{AE} = \frac{\cos^2(\phi - \theta - \psi)}{\cos\psi \cos^2\theta \cos(\delta + \theta + \psi) \left[ 1 + \frac{\sin(\delta + \phi)\sin(\phi - \beta - \psi)}{\cos(\delta + \theta + \psi)\cos(\beta - \theta)} \right]^2}$$

FIGURE 10.3 Coulomb’s earth pressure ($k_A$) equation for static conditions. Also shown is the Mononobe-Okabe equation ($k_{AE}$) for earthquake conditions. (Figure reproduced from NAVFAC DM-7.2, 1982, with equations from Kramer 1996.)
load-carrying members not be supported by the active wedge, or else they will be subjected to lateral movement. The active wedge is inclined at an angle of $45^\circ + \phi/2$ from the horizontal, as indicated in Fig. 10.4.

**Passive Earth Pressure.** As shown in Fig. 10.4, the passive earth pressure is developed along the front side of the footing. Passive pressure is developed when the wall footing moves laterally into the soil and a passive wedge is developed. To calculate the passive resultant force $P_p$, the following equation is used, assuming that there is cohesionless soil in front of the wall footing:

$$P_p = \frac{1}{2} k_p \gamma \gamma D^2$$

(10.3)

where $P_p$ = passive resultant force in kilonewtons per linear meter of wall or pounds per linear foot of wall, $k_p$ = passive earth pressure coefficient, $\gamma_i$ = total unit weight of the soil located in front of the wall footing, and $D$ = depth of the wall footing (vertical distance from the ground surface in front of the retaining wall to the bottom of the footing). The passive earth pressure coefficient $k_p$ is equal to

$$k_p = \tan^2 (45^\circ + \frac{1}{2} \phi)$$

(10.4)

where $\phi$ = friction angle of the soil in front of the wall footing. Equation (10.4) is known as the passive Rankine state. To develop passive pressure, the wall footing must move laterally into the soil. The wall translation to reach the passive state is at least twice that required to reach the active earth pressure state. Usually it is desirable to limit the amount of wall translation by applying a reduction factor to the passive pressure. A commonly used reduction factor is 2.0. The soil engineer routinely reduces the passive pressure by one-half (reduction factor = 2.0) and then refers to the value as the allowable passive pressure.

**Note:** For active and passive wedge development there must be movement of the retaining wall as illustrated above.

**FIGURE 10.4** Active wedge behind retaining wall.
Footing Bearing Pressure. To calculate the footing bearing pressure, the first step is to sum the vertical loads, such as the wall and footing weights. The vertical loads can be represented by a single resultant vertical force, per linear meter or foot of wall, that is offset by a distance (eccentricity) from the toe of the footing. This can then be converted to a pressure distribution by using Eq. (8.7). The largest bearing pressure is routinely at the toe of the footing, and it should not exceed the allowable bearing pressure (Sec. 8.2.5).

Retaining Wall Analyses. Once the active earth pressure resultant force $P_A$ and the passive resultant force $P_p$ have been calculated, the design analysis is performed as indicated in Fig. 10.2c. The retaining wall analysis includes determining the resultant location of the forces (i.e., calculate $d$, which should be within the middle third of the footing), the factor of safety for overturning, and the factor of safety for sliding. The adhesion $c$, between the bottom of the footing and the underlying soil is often ignored for the sliding analysis.

10.1.2 Retaining Wall Analyses for Earthquake Conditions

The performance of retaining walls during earthquakes is very complex. As stated by Kramer (1996), laboratory tests and analyses of gravity walls subjected to seismic forces have indicated the following:

1. Walls can move by translation and/or rotation. The relative amounts of translation and rotation depend on the design of the wall; one or the other may predominate for some walls, and both may occur for others (Nadim and Whitman 1984, Siddharthan et al. 1992).

2. The magnitude and distribution of dynamic wall pressures are influenced by the mode of wall movement, e.g., translation, rotation about the base, or rotation about the top (Sherif et al. 1982, Sherif and Fang 1984a, b).

3. The maximum soil thrust acting on a wall generally occurs when the wall has translated or rotated toward the backfill (i.e., when the inertial force on the wall is directed toward the backfill). The minimum soil thrust occurs when the wall has translated or rotated away from the backfill.

4. The shape of the earthquake pressure distribution on the back of the wall changes as the wall moves. The point of application of the soil thrust therefore moves up and down along the back of the wall. The position of the soil thrust is highest when the wall has moved toward the soil and lowest when the wall moves outward.

5. Dynamic wall pressures are influenced by the dynamic response of the wall and backfill and can increase significantly near the natural frequency of the wall-backfill system (Steedman and Zeng 1990). Permanent wall displacements also increase at frequencies near the natural frequency of the wall-backfill system (Nadim 1982). Dynamic response effects can also cause deflections of different parts of the wall to be out of phase. This effect can be particularly significant for walls that penetrate into the foundation soils when the backfill soils move out of phase with the foundation soils.

6. Increased residual pressures may remain on the wall after an episode of strong shaking has ended (Whitman 1990).

Because of the complex soil-structure interaction during the earthquake, the most commonly used method for the design of retaining walls is the pseudostatic method, which is discussed in Sec. 10.2.

10.1.3 One-Third Increase in Soil Properties for Seismic Conditions

When the recommendations for the allowable soil pressures at a site are presented, it is common practice for the geotechnical engineer to recommend that the allowable bearing pressure
and the allowable passive pressure be increased by a factor of one-third when performing seismic analyses. For example, in soil reports, it is commonly stated: “For the analysis of earthquake loading, the allowable bearing pressure and passive resistance may be increased by a factor of one-third.” The rationale behind this recommendation is that the allowable bearing pressure and allowable passive pressure have an ample factor of safety, and thus for seismic analyses, a lower factor of safety would be acceptable.

Usually the above recommendation is appropriate if the retaining wall bearing material and the soil in front of the wall (i.e., passive wedge area) consist of the following:

- Massive crystalline bedrock and sedimentary rock that remains intact during the earthquake.
- Soils that tend to dilate during the seismic shaking or, e.g., dense to very dense granular soil and heavily overconsolidated cohesive soil such as very stiff to hard clays.
- Soils that have a stress-strain curve that does not exhibit a significant reduction in shear strength with strain.
- Clay that has a low sensitivity.
- Soils located above the groundwater table. These soils often have negative pore water pressure due to capillary action.

These materials do not lose shear strength during the seismic shaking, and therefore an increase in bearing pressure and passive resistance is appropriate.

A one-third increase in allowable bearing pressure and allowable passive pressure should not be recommended if the bearing material and/or the soil in front of the wall (i.e., passive wedge area) consists of the following:

- Foliated or friable rock that fractures apart during the earthquake, resulting in a reduction in shear strength of the rock.
- Loose soil located below the groundwater table and subjected to liquefaction or a substantial increase in pore water pressure.
- Sensitive clays that lose shear strength during the earthquake.
- Soft clays and organic soils that are overloaded and subjected to plastic flow.

These materials have a reduction in shear strength during the earthquake. Since the materials are weakened by the seismic shaking, the static values of allowable bearing pressures and allowable passive resistance should not be increased for the earthquake analyses. In fact, the allowable bearing pressure and the allowable passive pressure may actually have to be reduced to account for the weakening of the soil during the earthquake. Sections 10.3 and 10.4 discuss retaining wall analyses for the case where the soil is weakened during the earthquake.

### 10.2 PSEUDOSTATIC METHOD

#### 10.2.1 Introduction

The most commonly used method of retaining wall analyses for earthquake conditions is the pseudostatic method. The pseudostatic method is also applicable for earthquake slope stability analyses (see Sec. 9.2). As previously mentioned, the advantages of this method are that it is easy to understand and apply.
Similar to earthquake slope stability analyses, this method ignores the cyclic nature of the earthquake and treats it as if it applied an additional static force upon the retaining wall. In particular, the pseudostatic approach is to apply a lateral force upon the retaining wall. To derive the lateral force, it can be assumed that the force acts through the centroid of the active wedge. The pseudostatic lateral force $P_e$ is calculated by using Eq. (6.1), or

$$P_e = ma = \frac{W}{g} a = W \frac{a_{\text{max}}}{g} = k_h W$$

(10.5)

where $P_e =$ horizontal pseudostatic force acting upon the retaining wall, lb or kN. This force can be assumed to act through the centroid of the active wedge. For retaining wall analyses, the wall is usually assumed to have a unit length (i.e., two-dimensional analysis)

$m =$ total mass of active wedge, lb or kg, which is equal to $W/g$

$W =$ total weight of active wedge, lb or kN

$a =$ acceleration, which in this case is maximum horizontal acceleration at ground surface caused by the earthquake ($a = a_{\text{max}}$), ft/s$^2$ or m/s$^2$

$a_{\text{max}} =$ maximum horizontal acceleration at ground surface that is induced by the earthquake, ft/s$^2$ or m/s$^2$. The maximum horizontal acceleration is also commonly referred to as the peak ground acceleration (see Sec. 5.6)

$$a_{\text{max}}/g = k_h = \text{seismic coefficient, also known as pseudostatic coefficient (dimensionless)}$$

Note that an earthquake could subject the active wedge to both vertical and horizontal pseudostatic forces. However, the vertical force is usually ignored in the standard pseudostatic analysis. This is because the vertical pseudostatic force acting on the active wedge usually has much less effect on the design of the retaining wall. In addition, most earthquakes produce a peak vertical acceleration that is less than the peak horizontal acceleration, and hence $k_v$ is smaller than $k_h$.

As indicated in Eq. (10.5), the only unknowns in the pseudostatic method are the weight of the active wedge $W$ and the seismic coefficient $k_h$. Because of the usual relatively small size of the active wedge, the seismic coefficient $k_h$ can be assumed to be equal to $a_{\text{max}}/g$.

Using Fig. 10.4, the weight of the active wedge can be calculated as follows:

$$W = \frac{1}{2} HL \gamma_i = \frac{1}{2} H [H \tan (45^\circ - \frac{1}{2} \phi)] \gamma_i = \frac{1}{2} k_h^{1/2} H^2 \gamma_i$$

(10.6)

where $W =$ weight of the active wedge, lb or kN per unit length of wall

$H =$ height of the retaining wall, ft or m

$L =$ length of active wedge at top of retaining wall. Note in Fig. 10.4 that the active wedge is inclined at an angle equal to $45^\circ + \frac{1}{2} \phi$. Therefore the internal angle of the active wedge is equal to $90^\circ - (45^\circ + \frac{1}{2} \phi) = 45^\circ - \frac{1}{2} \phi$. The length $L$ can then be calculated as $L = H \tan (45^\circ - \frac{1}{2} \phi) = H k_h^{1/2}$

$\gamma_i =$ total unit weight of the backfill soil (i.e., unit weight of soil comprising active wedge), lb/ft$^3$ or kN/m$^3$

Substituting Eq. (10.6) into Eq. (10.5), we get for the final result:

$$P_e = k_h W = \frac{1}{2} k_h k_h^{1/2} H^2 \gamma_i = \frac{1}{2} k_h^{3/2} \left( \frac{a_{\text{max}}}{g} \right) (H^2 \gamma_i)$$

(10.7)

Note that since the pseudostatic force is applied to the centroid of the active wedge, the location of the force $P_e$ is at a distance of $\frac{1}{2}H$ above the base of the retaining wall.
10.2.2 Method by Seed and Whitman

Seed and Whitman (1970) developed an equation that can be used to determine the horizontal pseudostatic force acting on the retaining wall:

\[
P_e = \frac{3}{8} \frac{a_{\text{max}}}{g} H^2 \gamma_i \tag{10.8}
\]

Note that the terms in Eq. (10.8) have the same definitions as the terms in Eq. (10.7). Comparing Eqs. (10.7) and (10.8), we see the two equations are identical for the case where \(\frac{1}{2} k_A^{1/2} = h\). According to Seed and Whitman (1970), the location of the pseudostatic force from Eq. (10.8) can be assumed to act at a distance of 0.6H above the base of the wall.

10.2.3 Method by Mononobe and Okabe

Mononobe and Matsuo (1929) and Okabe (1926) also developed an equation that can be used to determine the horizontal pseudostatic force acting on the retaining wall. This method is often referred to as the Mononobe-Okabe method. The equation is an extension of the Coulomb approach and is

\[
P_{AE} = P_A + P_E = \frac{1}{2} k_{AE} H^2 \gamma_i \tag{10.9}
\]

where \(P_{AE}\) is the sum of the static \((P_A)\) and the pseudostatic earthquake force \((P_E)\). The equation for \(k_{AE}\) is shown in Fig. 10.3. Note that in Fig. 10.3, the term \(\psi\) is defined as

\[
\psi = \tan^{-1} k_A = \tan^{-1} \frac{a_{\text{max}}}{g} \tag{10.10}
\]

The original approach by Mononobe and Okabe was to assume that the force \(P_{AE}\) from Eq. (10.9) acts at a distance of \(h/3\) above the base of the wall.

10.2.4 Example Problem

Figure 10.5 (from Lambe and Whitman 1969) presents an example of a proposed concrete retaining wall that will have a height of 20 ft (6.1 m) and a base width of 7 ft (2.1 m). The wall will be backfilled with sand that has a total unit weight \(\gamma_i\) of 110 lb/ft\(^3\) (17.3 kN/m\(^3\)), friction angle \(\phi\) of 30\(^\circ\), and an assumed wall friction \(\delta = \phi_w\) of 30\(^\circ\). Although \(\phi = 30\(^\circ\)\) is used for this example problem, more typical values of wall friction are \(\phi_w = \frac{1}{2} \phi\) for the wall friction between granular soil and wood or concrete walls, and \(\phi_w = 20\(^\circ\)\) for the wall friction between granular soil and steel walls such as sheet pile walls. The retaining wall is analyzed for the static case and for the earthquake condition assuming \(k_a = 0.2\). It is also assumed that the backfill soil, bearing soil, and soil located in the passive wedge are not weakened by the earthquake.

**Static Analysis**

*Active Earth Pressure.* For the example problem shown in Fig. 10.5, the value of the active earth pressure coefficient \(k_a\) can be calculated by using Coulomb’s equation (Fig. 10.3) and inserting the following values:

- Slope inclination: \(\beta = 0\) (no slope inclination)
- Back face of the retaining wall: \(\theta = 0\) (vertical back face of the wall)
FIGURE 10.5a  Example problem. Cross section of proposed retaining wall and resultant forces acting on the retaining wall. (From Lambe and Whitman 1969; reproduced with permission of John Wiley & Sons.)
Friction between the back face of the wall and the soil backfill: \( \delta = \phi_w = 30^\circ \)

Friction angle of backfill sand: \( \phi = 30^\circ \)

Inputting the above values into Coulomb's equation (Fig. 10.3), the value of the active earth pressure coefficient \( k_A = 0.297 \).

By using Eq. (10.1) with \( k_A = 0.297 \), total unit weight \( \gamma = 110 \text{ lb/ft}^3 (17.3 \text{ kN/m}^3) \), and the height of the retaining wall \( H = 20 \text{ ft} \) (see Fig. 10.5a), the active earth pressure resultant force \( P_A = 6540 \text{ lb per linear foot of wall} \) (95.4 kN per linear meter of wall). As indicated in Fig. 10.5a, the active earth pressure resultant force \( P_A = 6540 \text{ lb/ft} \) is inclined at an angle of 30° due to the wall friction assumptions. The vertical \( P_v = 3270 \text{ lb/ft} \) and horizontal \( P_h = 5660 \text{ lb/ft} \) resultants of \( P_A \) are also shown in Fig. 10.5a. Note in Fig. 10.3 that even...
Next the bearing stress is computed. The average bearing stress is $15,270/7 = 2180$ psf. Assuming that the bearing stress is distributed linearly, the maximum stress can be found

$$
\sigma_{\text{mom}} = \frac{M}{S}
$$

where

$$
M = \text{moment about } A = 15,270(3.5 - 2.66) = 12,820 \text{ lb-ft/ft}
$$

$$
S = \text{section modulus} = \frac{1}{6}B^2 = \frac{1}{6}(7)^2 = 8.17 \text{ ft}^3
$$

where $B$ is width of base.

$$
\sigma_{\text{mom}} = \frac{12,820}{8.17} = 1570 \text{ psf}
$$

Maximum stress = $2180 + 1570 = 3750 \text{ psf}$

Finally, the resistance to horizontal sliding is checked. Assuming passive resistance without wall friction,

$$
K_s = 3
$$

$$
P_a = \frac{1}{3}(110)(3^3)(3) = 1500 \text{ lb/ft}
$$

With reduction factor of 2,

$$
P_a = \frac{1}{2}1500 = 750 \text{ lb/ft}
$$

$$
T = 5660 - 750 = 4910 \text{ lb/ft}
$$

$$
N \tan 30^\circ = 8810 \text{ lb/ft}
$$

Ignoring passive resistance

$$
\frac{N \tan \phi_{ev}}{T} = 1.79 < 2 \quad \text{not OK}
$$

$$
\frac{N \tan \phi_{cr}}{T} = 1.55 > 1.5 \quad \text{OK}
$$

**FIGURE 10.5c** Example problem (continued). Calculation of the maximum bearing stress and the factor of safety for sliding. (From Lambe and Whitman 1969, reproduced with permission of John Wiley & Sons.)
**Footing Bearing Pressure.** The procedure for the calculation of the footing bearing pressure is as follows:

1. **Calculate N:** As indicated in Fig. 10.5b, the first step is to calculate $N (15,270 \text{ lb/ft})$, which equals the sum of the weight of the wall, footing, and vertical component of the active earth pressure resultant force (that is, $N = W + P_a \sin \phi_v$).

2. **Determine resultant location of N:** The resultant location of $N$ from the toe of the retaining wall (that is, 2.66 ft) is calculated as shown in Fig. 10.5b. The moments are determined about the toe of the retaining wall. Then the location of $N$ is equal to the difference in the opposing moments divided by $N$.

3. **Determine average bearing pressure:** The average bearing pressure ($2180 \text{ lb/ft}^2$) is calculated in Fig. 10.5c as $N$ divided by the width of the footing (7 ft).

4. **Calculate moment about the centerline of the footing:** The moment about the centerline of the footing is calculated as $N$ times the eccentricity (0.84 ft).

5. **Section modulus:** The section modulus of the footing is calculated as shown in Fig. 10.5c.

6. **Portion of bearing stress due to moment:** The portion of the bearing stress due to the moment ($\sigma_{mom} = 1570 \text{ lb/ft}^2$).

7. **Maximum bearing stress:** The maximum bearing stress is then calculated as the sum of the average stress ($\sigma_{avg} = 2180 \text{ lb/ft}^2$) plus the bearing stress due to the moment ($\sigma_{mom} = 1570 \text{ lb/ft}^2$).

As indicated in Fig. 10.5c, the maximum bearing stress is 3750 lb/ft$^2$ (180 kPa). This maximum bearing stress must be less than the allowable bearing pressure (Chap. 8). It is also a standard requirement that the resultant normal force $N$ be located within the middle third of the footing, such as illustrated in Fig. 10.5b. As an alternative to the above procedure, Eq. (8.7) can be used to calculate the maximum and minimum bearing stress.

**Sliding Analysis.** The factor of safety (FS) for sliding of the retaining wall is often defined as the resisting forces divided by the driving force. The forces are per linear meter or foot of wall, or

$$FS = \frac{N \tan \delta + P_e}{P_h}$$  \hspace{1cm} (10.11)

where $\delta = \phi_v$ = friction angle between the bottom of the concrete foundation and bearing soil; $N$ = sum of the weight of the wall, footing, and vertical component of the active earth pressure resultant force (or $N = W + P_a \sin \phi_v$); $P_e$ = allowable passive resultant force [$P_e$ from Eq. (10.3) divided by a reduction factor]; and $P_h$ = horizontal component of the active earth pressure resultant force ($P_h = P_a \cos \phi_v$).

There are variations of Eq. (10.11) that are used in practice. For example, as illustrated in Fig. 10.5c, the value of $P_e$ is subtracted from $P_h$ in the denominator of Eq. (10.11), instead of $P_e$ being used in the numerator. For the example problem shown in Fig. 10.5, the factor of safety for sliding is $FS = 1.79$ when the passive pressure is included and $FS = 1.55$ when the passive pressure is excluded. For static conditions, the typical recommendations for minimum factor of safety for sliding are 1.5 to 2.0 (Cernica 1995b).

**Overturning Analysis.** The factor of safety for overturning of the retaining wall is calculated by taking moments about the toe of the footing and is

$$FS = \frac{W_a}{\sqrt{P_h H - P_e}}$$  \hspace{1cm} (10.12)
where $a$ = lateral distance from the resultant weight $W$ of the wall and footing to the toe of the footing, $P_h$ = horizontal component of the active earth pressure resultant force, $P_v$ = vertical component of the active earth pressure resultant force, and $e$ = lateral distance from the location of $P_v$ to the toe of the wall. In Fig. 10.5b, the factor of safety (ratio) for overturning is calculated to be 3.73. For static conditions, the typical recommendations for minimum factor of safety for overturning are 1.5 to 2.0 (Cernica 1995b).

**Settlement and Stability Analysis.** Although not shown in Fig. 10.5, the settlement and stability of the ground supporting the retaining wall footing should also be determined. To calculate the settlement and evaluate the stability for static conditions, standard settlement and slope stability analyses can be utilized (see chaps. 9 and 13, Day 2000).

**Earthquake Analysis.** The pseudostatic analysis is performed for the three methods outlined in Secs. 10.2.1 to 10.2.3.

**Equation (10.7).** Using Eq. (10.2) and neglecting the wall friction, we find

$$k_a = \tan^2(45^\circ - \frac{\phi}{2}) = \tan^2(45^\circ - \frac{\frac{1}{2}30^\circ}{2}) = 0.333$$

Substituting into Eq. (10.7) gives

$$P_e = \frac{1}{2} k_a \left( \frac{a_{\max}}{g} \right) (H^2 \gamma)$$

$$= \frac{1}{2} (0.333)^{1/2} (0.2) (20 \text{ ft})^2 (110 \text{ lb/ft}^3) = 2540 \text{ lb per linear foot of wall length}$$

This pseudostatic force acts at a distance of $\frac{1}{2}H$ above the base of the wall, or $\frac{1}{2}H = \frac{1}{2}(20 \text{ ft}) = 13.3 \text{ ft}$. Similar to Eq. (10.11), the factor of safety for sliding is

$$FS = \frac{N \tan \delta + P_e}{P_h + P_v}$$

(10.13)

Substituting values into Eq. (10.13) gives

$$FS = \frac{15.270 \tan 30^\circ + 750}{5660 + 2540} = 1.17$$

Based on Eq. (10.12), the factor of safety for overturning is

$$FS = \frac{W_a}{\frac{1}{2}P_hH - P_e + \frac{1}{2}HP_e}$$

(10.14)

Inserting values into Eq. (10.14) yields

$$FS = \frac{55,500}{\frac{1}{2}(5660)(20) - 3270(7) + \frac{1}{2}(20)(2540)} = 1.14$$

**Method by Seed and Whitman (1970).** Using Eq. (10.8) and neglecting the wall friction, we get

$$P_e = \frac{3}{8} \left( \frac{a_{\max}}{g} \right) H^2 \gamma$$

$$= \frac{1}{2} (0.2) (20 \text{ ft})^2 (110 \text{ lb/ft}^3) = 3300 \text{ lb per linear foot of wall length}$$

This pseudostatic force acts at a distance of $0.6H$ above the base of the wall, or $0.6H = (0.6)(20 \text{ ft}) = 12 \text{ ft}$. Using Eq. (10.13) gives
Similar to Eq. (10.14), the factor of safety for overturning is

$$FS = \frac{N \tan \delta + P_e}{P_H + P_E} = \frac{15,270 \tan 30^\circ + 750}{5660 + 3300} = 1.07$$

Substituting values into Eq. (10.15) gives

$$FS = \frac{W_a}{\sqrt{P_H H - P_E + 0.6HP_E}} \tag{10.15}$$

Substituting values into Eq. (10.15) gives

$$FS = \frac{55,500}{\sqrt{(5660)(20) - 3270(7) + 0.6(20)(3300)}} = 1.02$$

**Mononobe-Okabe Method.** We use the following values:

- $\theta$ (wall inclination) = 0°
- $\phi$ (friction angle of backfill soil) = 30°
- $\beta$ (backfill slope inclination) = 0°
- $\delta = \phi_w$ (friction angle between the backfill and wall) = 30°
- $\psi = \tan^{-1} k_s = \tan^{-1} \frac{a_{max}}{g} = \tan^{-1} 0.2 = 11.3^\circ$

Inserting the above values into the $K_{AE}$ equation in Fig. 10.3, we get $K_{AE} = 0.471$. Therefore, using Eq. (10.9) yields

$$P_{AE} = P_A + P_E = \frac{1}{2}k_{AE}H^2\gamma_i$$

$$= \frac{1}{2}(0.471)(20)^2(110) = 10,400 \text{ lb per linear foot of wall length}$$

This force $P_{AE}$ is inclined at an angle of 30° and acts at a distance of 0.33$H$ above the base of the wall, or 0.33$H$ = (0.33)(20 ft) = 6.67 ft. The factor of safety for sliding is

$$FS = \frac{N \tan \delta + P_e}{P_H + P_E} = \frac{(W + P_{AE} \sin \phi_w) \tan \delta + P_E}{P_{AE} \cos \phi_w} \tag{10.16}$$

Substituting values into Eq. (10.16) gives

$$FS = \frac{(3000 + 9000 + 10,400 \sin 30^\circ)(\tan 30^\circ) + 750}{10,400 \cos 30^\circ} = 1.19$$

The factor of safety for overturning is

$$FS = \frac{W_a}{\sqrt{H P_{AE} \cos \phi_w - P_E \sin \phi_w}} \tag{10.17}$$

Substituting values into Eq. (10.17) produces

$$FS = \frac{55,500}{\sqrt{(20)(10,400)(\cos 30^\circ) - (10,400)(\sin 30^\circ)(7)}} = 2.35$$
Summary of Values. The values from the static and earthquake analyses using $k_n = a_{max}/g = 0.2$ are summarized below:

<table>
<thead>
<tr>
<th>Type of condition</th>
<th>$P_E$ or $P_{AE}$, lb/ft</th>
<th>Location of $P_E$ or $P_{AE}$ above base of wall, ft</th>
<th>Factor of safety for sliding</th>
<th>Factor of safety for overturning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>$P_E = 0$</td>
<td>—</td>
<td>1.69*</td>
<td>3.73</td>
</tr>
<tr>
<td>Equation (10.7)</td>
<td>$P_E = 2,540$</td>
<td>$\frac{2}{3}H = 13.3$</td>
<td>1.17</td>
<td>1.14</td>
</tr>
<tr>
<td>Earthquake $(k_n = 0.2)$</td>
<td>$P_E = 3,300$</td>
<td>$0.6H = 12$</td>
<td>1.07</td>
<td>1.02</td>
</tr>
<tr>
<td>Whitman</td>
<td>$P_{AE} = 10,400$</td>
<td>$\frac{2}{3}H = 6.7$</td>
<td>1.19</td>
<td>2.35</td>
</tr>
</tbody>
</table>

*Factor of safety for sliding using Eq. (10.11).

For the analysis of sliding and overturning of the retaining wall, it is common to accept a lower factor of safety (1.1 to 1.2) under the combined static and earthquake loads. Thus the retaining wall would be considered marginally stable for the earthquake sliding and overturning conditions.

Note in the above table that the factor of safety for overturning is equal to 2.35 based on the Mononobe-Okabe method. This factor of safety is much larger than that for the other two methods. This is because the force $P_{AE}$ is assumed to be located at a distance of $\frac{2}{3}H$ above the base of the wall. Kramer (1996) suggests that it is more appropriate to assume that $P_E$ is located at a distance of $0.6H$ above the base of the wall [that is, $P_E / P_{AE} = P_E$, see Eq. (10.9)].

Although the calculations are not shown, it can be demonstrated that the resultant location of $N$ for the earthquake condition is outside the middle third of the footing. Depending on the type of material beneath the footing, this condition could cause a bearing capacity failure or excess settlement at the toe of the footing during the earthquake.

10.2.5 Mechanically Stabilized Earth Retaining Walls

Introduction. Mechanically stabilized earth (MSE) retaining walls are typically composed of strip- or grid-type (geosynthetic) reinforcement. Because they are often more economical to construct than conventional concrete retaining walls, mechanically stabilized earth retaining walls have become very popular in the past decade.

A mechanically stabilized earth retaining wall is composed of three elements: (1) wall facing material, (2) soil reinforcement, such as strip- or grid-type reinforcement, and (3) compacted fill between the soil reinforcement. Figure 10.6 shows the construction of a mechanically stabilized earth retaining wall.

The design analyses for a mechanically stabilized earth retaining wall are more complex than those for a cantilevered retaining wall. For a mechanically stabilized earth retaining wall, both the internal and external stability must be checked, as discussed below.

External Stability—Static Conditions. The analysis for the external stability is similar to that for a gravity retaining wall. For example, Figs. 10.7 and 10.8 present the design analysis for external stability for a level backfill condition and a sloping backfill condition. In both
Figs. 10.7 and 10.8, the zone of mechanically stabilized earth mass is treated in a similar fashion as a massive gravity retaining wall. For static conditions, the following analyses must be performed:

1. **Allowable bearing pressure**: The bearing pressure due to the reinforced soil mass must not exceed the allowable bearing pressure.
2. **Factor of safety for sliding:** The reinforced soil mass must have an adequate factor of safety for sliding.

3. **Factor of safety for overturning:** The reinforced soil mass must have an adequate factor of safety for overturning about point $O$.

4. **Resultant of vertical forces:** The resultant of the vertical forces $N$ must be within the middle one-third of the base of the reinforced soil mass.

5. **Stability of reinforced soil mass:** The stability of the entire reinforced soil mass (i.e., shear failure below the bottom of the wall) should be checked.

   Note in Fig. 10.7 that two forces $P_1$ and $P_2$ are shown acting on the reinforced soil mass. The first force $P_1$ is determined from the standard active earth pressure resultant equation [Eq. (10.1)]. The second force $P_2$ is due to a uniform surcharge $Q$ applied to the entire ground surface behind the mechanically stabilized earth retaining wall. If the wall does not have a surcharge, then $P_2$ is equal to zero.

   Figure 10.8 presents the active earth pressure force for an inclined slope behind the retaining wall. As shown in Fig. 10.8, the friction $\delta$ of the soil along the backside of the reinforced soil mass has been included in the analysis. The value of $k_A$ would be obtained.

---

**FIGURE 10.8** Static design analysis for mechanically stabilized earth retaining wall having sloping backfill. (Adapted from Standard Specifications for Highway Bridges, AASHTO 1996.)
from Coulomb’s earth pressure equation (Fig. 10.3). As a conservative approach, the friction angle \(\delta\) can be assumed to be equal to zero, and then \(P_H = P_A\). As indicated in both Figs. 10.7 and 10.8, the minimum width of the reinforced soil mass must be at least \(\frac{7}{10}\) times the height of the reinforced soil mass.

**External Stability—Earthquake Conditions.** For earthquake conditions, the most commonly used approach is the pseudostatic method. The pseudostatic force can be calculated from Eqs. (10.7), (10.8), or (10.9). Once the pseudostatic force and location are known, then the five items listed in “External Stability—Static Conditions” would need to be checked. Acceptable values of the factors of safety for sliding and overturning are typically in the range of 1.1 to 1.2 for earthquake conditions.

**Internal Stability.** To check the static stability of the mechanically stabilized zone, a slope stability analysis can be performed in which the soil reinforcement is modeled as horizontal forces equivalent to its allowable tensile resistance. For earthquake conditions, the slope stability analysis could incorporate a pseudostatic force (i.e., Sec. 9.2.4). In addition to calculating the factor of safety for both the static and earthquake conditions, the pullout resistance of the reinforcement along the slip surface should be checked.

**Example Problem.** Using the mechanically stabilized earth retaining wall shown in Fig. 10.7, let \(H = 20\) ft, the width of the mechanically stabilized earth retaining wall = 14 ft, the depth of embedment at the front of the mechanically stabilized zone = 3 ft, and there is a level backfill with no surcharge pressures (that is, \(P_s = 0\)). Assume that the soil behind and in front of the mechanically stabilized zone is a clean sand having a friction angle \(\phi = 30^\circ\), a total unit weight of \(\gamma_i = 110\) lb/ft\(^3\), and there will be no shear stress (that is, \(\delta = 0^\circ\)) along the vertical back and front sides of the mechanically stabilized zone. For the mechanically stabilized zone, assume the soil will have a total unit weight \(\gamma_i = 120\) lb/ft\(^3\) and \(\delta = 23^\circ\) along the bottom of the mechanically stabilized zone. For earthquake design conditions, use \(a_{\text{max}} = 0.20g\). Calculate the factor of safety for sliding and for overturning for both the static and earthquake conditions.

**Solution: Static Analysis**

\[
k_a = \tan^2 (45^\circ - \frac{1}{2} \phi) = \tan^2 [45^\circ - \frac{1}{2}(30^\circ)] = 0.333
\]

\[
k_p = \tan^2 (45^\circ + \frac{1}{2} \phi) = \tan^2 [45^\circ + \frac{1}{2}(30^\circ)] = 3.0
\]

\[
P_A = \frac{1}{2} k_a \gamma_i H^2 = \frac{1}{2}(0.333)(110)(20)^2 = 7330 \text{ lb/ft}
\]

\[
P_p = \frac{1}{2} k_p \gamma_i D^2 = \frac{1}{2} (3.0)(110)(3)^2 = 1490 \text{ lb/ft}
\]

With reduction factor = 2,

Allowable \(P_p = 740\) lb/ft

For sliding analysis:

\[
FS = N \tan \delta + \frac{P_r}{P_A} \quad \text{Eq. (10.11)}, \quad \text{where } P_A = P_H
\]

\[
W = N = H L \gamma_i = (20)(14)(120 \text{ lb/ft}^3) = 33,600 \text{ lb per linear foot of wall length}
\]
For overturning analysis: Taking moments about the toe of the wall gives

\[
\text{Overturning moment} = \frac{P_v H}{3} = \frac{7330 \times 20}{3} = 48,900
\]

\[
\text{Moment of weight} = 33,600 \times \frac{14}{2} = 235,000
\]

\[
\text{FS} = \frac{235,000}{48,900} = 4.81
\]

**Solution: Earthquake Analysis.** Using Eq. (10.7), we get

\[
P_e = \frac{1}{2} k_{ih}^2 \left( \frac{a_{\max}}{g} \right) (H^2 \gamma_h) = \frac{1}{2} (0.333)^{1/2} (0.20)(20)^2(1)(110) = 2540 \text{ lb/ft}
\]

For sliding analysis, use Eq. (10.13):

\[
\text{FS} = \frac{N \tan \delta + P_e}{P_h + P_E} = \frac{33,600 \tan 23^\circ + 740}{7330 + 2540} = 1.52
\]

For overturning analysis, use Eq. (10.14) with \( P_e = 0 \).

\[
\text{FS} = \frac{W_h}{\frac{1}{2} P_h H + \frac{1}{2} H P_E} = \frac{33,600(7)}{\frac{1}{2} (7330)(20) + \frac{1}{2}(20)(2540)} = 2.84
\]

In summary,

**Static conditions:**

\[
\text{FS sliding} = 2.05
\]

\[
\text{FS overturning} = 4.81
\]

**Earthquake conditions (\( a_{\max} = 0.20g \)):**

\[
\text{FS sliding} = 1.52
\]

\[
\text{FS overturning} = 2.84
\]
as seawalls, anchored bulkheads, gravity and cantilever walls, and sheet pile cofferdams, that allow large ships to moor adjacent to the retaining walls and then load or unload cargo. Examples of liquefaction-induced damage to retaining walls are presented in Sec. 3.4.3.

There are often three different types of liquefaction effects that can damage the retaining wall:

1. **Passive wedge liquefaction:** The first is liquefaction of soil in front of the retaining wall. In this case, the passive resistance in front of the retaining wall is reduced.

2. **Active wedge liquefaction:** In the second case, the soil behind the retaining wall liquefies, and the pressure exerted on the wall is greatly increased. Cases 1 and 2 can act individually or together, and they can initiate an overturning failure of the retaining wall or cause the wall to progressively slide outward (localized lateral spreading) or tilt toward the water. Another possibility is that the increased pressure exerted on the wall could exceed the strength of the wall, resulting in a structural failure of the wall.

Liquefaction of the soil behind the retaining wall can also affect tieback anchors. For example, the increased pressure due to liquefaction of the soil behind the wall could break the tieback anchors or reduce their passive resistance.

3. **Liquefaction below base of wall:** The third case is liquefaction below the bottom of the wall. Many waterfront retaining walls consist of massive structures, such as the concrete box caissons shown in Fig. 3.31. In this case, the bearing capacity or slide resistance of the wall is reduced, resulting in a bearing capacity failure or promoting lateral spreading of the wall.

### 10.3.2 Design Pressures

The first step in the analysis is to determine the factor of safety against liquefaction for the soil behind the retaining wall, in front of the retaining wall, and below the bottom of the wall. The analysis presented in Chap. 6 can be used to determine the factor of safety against liquefaction. The retaining wall may exert significant shear stress into the underlying soil, which can decrease the factor of safety against liquefaction for loose soils (i.e., see Fig. 9.24). Likewise, there could be sloping ground in front of the wall or behind the wall, in which case the factor of safety against liquefaction may need to be adjusted (see Sec. 9.4.2).

After the factor of safety against liquefaction has been calculated, the next step is to determine the design pressures that act on the retaining wall:

1. **Passive pressure:** For those soils that will be subjected to liquefaction in the passive zone, one approach is to assume that the liquefied soil has zero shear strength. In essence, the liquefied zones no longer provide sliding or overturning resistance.

2. **Active pressure:** For those soils that will be subjected to liquefaction in the active zone, the pressure exerted on the face of the wall will increase. One approach is to assume zero shear strength of the liquefied soil (that is, \( \phi^* = 0 \)). There are two possible conditions:

   a. **Water level located only behind the retaining wall:** In this case, the wall and the ground beneath the bottom of the wall are relatively impermeable. In addition, there is a groundwater table behind the wall with dry conditions in front of the wall. The thrust on the wall due to liquefaction of the backfill can be calculated by using Eq. (10.1) with \( k_A = 1 \) [i.e., for \( \phi^* = 0, k_A = 1 \), see Eq. (10.2)] and \( \gamma_s = \gamma_{sat} \) (i.e., \( \gamma_{sat} = \) saturated unit weight of the soil).

   b. **Water levels are approximately the same on both sides of the retaining wall:** The more common situation is that the elevation of the groundwater table behind the wall is approximately the same as the water level in front of the wall. The thrust on the wall due to liquefaction of the soil can be calculated by using Eq. (10.1) with \( k_A = 1 \) [i.e., for \( \phi^* = 0, k_A = 1 \), see Eq. (10.2)] and using \( \gamma_b \) (buoyant unit weight) in place of \( \gamma_s \).
The only difference between the two cases is that the first case includes the unit weight of water \((\gamma_w = \gamma_s + \gamma_b)\), while the second case does not include \(\gamma_w\) because it is located on both sides of the wall and hence its effect is canceled out.

In addition to the increased pressure acting on the retaining wall due to liquefaction, consider a reduction in support and/or resistance of the tieback anchors.

3. **Bearing soil:** For the liquefaction of the bearing soil, use the analysis in Sec. 8.2.

### 10.3.3 Sheet Pile Walls

**Introduction.** Sheet pile retaining walls are widely used for waterfront construction and consist of interlocking members that are driven into place. Individual sheet piles come in many different sizes and shapes. Sheet piles have an interlocking joint that enables the individual segments to be connected together to form a solid wall.

**Static Design.** Many different types of static design methods are used for sheet pile walls. Figure 10.9 shows the most common type of static design method. In Fig. 10.9, the term \(H\) represents the unsupported face of the sheet pile wall. As indicated in Fig. 10.9, this sheet pile wall is being used as a waterfront retaining structure, and the elevation of the water in front of the wall is the same as that of the groundwater table behind the wall. For highly permeable soil, such as clean sand and gravel, this often occurs because the water can quickly flow underneath the wall in order to equalize the water levels.

In Fig. 10.9, the term \(D\) represents that portion of the sheet pile wall that is anchored in soil. Also shown in Fig. 10.9 is a force designated as \(A_p\). This represents a restraining force on the sheet pile wall due to the construction of a tieback, such as by using a rod that has a

![Earth pressure diagram for static design of sheet pile wall.](From NAVFAC DM-7.2, 1982.)
grouted end or is attached to an anchor block. Tieback anchors are often used in sheet pile wall construction to reduce the bending moments in the sheet pile. When tieback anchors are used, the sheet pile wall is typically referred to as an anchored bulkhead, while if no tiebacks are utilized, the wall is called a cantilevered sheet pile wall.

Sheet pile walls tend to be relatively flexible. Thus, as indicated in Fig. 10.9, the design is based on active and passive earth pressures. The soil behind the wall is assumed to exert an active earth pressure on the sheet pile wall. At the groundwater table (point A), the active earth pressure is equal to

$$\text{Active earth pressure at point } A, \text{ kPa or lb/ft}^2 = k_A \gamma_1 d_1$$  \hspace{1cm} (10.18)

where $k_A$ = active earth pressure coefficient from Eq. (10.2) (dimensionless parameter).

Friction between sheet pile wall and soil is usually neglected in design analysis.

$$\gamma_1 = \text{total unit weight of the soil above the groundwater table, kN/m}^3 \text{ or lb/ft}^3$$

$$d_1 = \text{depth from the ground surface to the groundwater table, m or ft}$$

In using Eq. (10.18), a unit length (1 m or 1 ft) of sheet pile wall is assumed. At point B in Fig. 10.9, the active earth pressure equals

$$\text{Active earth pressure at point } B, \text{ kPa or lb/ft}^2 = k_A \gamma_1 d_1 + k_b \gamma_b d_2$$ \hspace{1cm} (10.19)

where $\gamma_b$ = buoyant unit weight of the soil below the groundwater table and $d_2$ = depth from the groundwater table to the bottom of the sheet pile wall. For a sheet pile wall having assumed values of $H$ and $D$ (see Fig. 10.9) and using the calculated values of active earth pressure at points A and B, the active earth pressure resultant force $P_A$, in kilonewtons per linear meter of wall or pounds per linear foot of wall, can be calculated.

The soil in front of the wall is assumed to exert a passive earth pressure on the sheet pile wall. The passive earth pressure at point C in Fig. 10.9 is

$$\text{Passive earth pressure at point } C, \text{ kPa or lb/ft}^2 = k_p \gamma_1 D$$ \hspace{1cm} (10.20)

where the passive earth pressure coefficient $k_p$ can be calculated from Eq. (10.4). Similar to the analysis of cantilever retaining walls, if it is desirable to limit the amount of sheet pile wall translation, then a reduction factor can be applied to the passive pressure. Once the allowable passive pressure is known at point C, the passive resultant force $P_p$ can be readily calculated.

As an alternative solution for the passive pressure, Eq. (10.3) can be used to calculate $P_p$ with the buoyant unit weight $\gamma_b$ substituted for the total unit weight $\gamma_1$ and the depth $D$ as shown in Fig. 10.9.

Note that a water pressure has not been included in the analysis. This is because the water level is the same on both sides of the wall, and water pressure cancels and thus should not be included in the analysis.

The static design of sheet pile walls requires the following analyses: (1) evaluation of the earth pressures that act on the wall, such as shown in Fig. 10.9; (2) determination of the required depth $D$ of piling penetration; (3) calculation of the maximum bending moment $M_{max}$, which is used to determine the maximum stress in the sheet pile; and (4) selection of the appropriate piling type, size, and construction details.

A typical design process is to assume a depth $D$ (Fig. 10.9) and then calculate the factor of safety for toe failure (i.e., toe kick-out) by the summation of moments at the tieback anchor (point D). The factor of safety is defined as the moment due to the passive force divided by the moment due to the active force. Values of acceptable FS for toe failure are 2 to 3. An alternative solution is to first select the factor of safety and then develop the active and passive resultant forces and moment arms in terms of $D$. By solving the equation, the value of $D$ for a specific factor of safety can be directly calculated.
Once the depth $D$ of the sheet pile wall is known, the anchor pull $A_p$ must be calculated. The anchor pull is determined by the summation of forces in the horizontal direction, or

$$A_p = P_a - \frac{P_p}{FS}$$  \hspace{1cm} (10.21)

where $P_a$ and $P_p$ are the resultant active and passive forces (see Fig. 10.9) and FS is the factor of safety that was obtained from the toe failure analysis. Based on the earth pressure diagram (Fig. 10.9) and the calculated value of $A_p$, elementary structural mechanics can be used to determine the maximum moment in the sheet pile wall. The maximum moment divided by the section modulus can then be compared with the allowable design stresses of the sheet piling.

Some other important design considerations for the static design of sheet pile walls include the following:

1. **Soil layers**: The active and passive earth pressures should be adjusted for soil layers having different engineering properties.

2. **Penetration depth**: The penetration depth $D$ of the sheet pile wall should be increased by at least an additional 20 percent to allow for the possibility of dredging and scour. Deeper penetration depths may be required based on a scour analysis.

3. **Surcharge loads**: The ground surface behind the sheet pile wall is often subjected to surcharge loads. The equation $P_s = QHkA$ can be used to determine the active earth pressure resultant force due to a uniform surcharge pressure applied to the ground surface behind the wall. Note in this equation that the entire height of the sheet pile wall (that is, $H + D$, see Fig. 10.9) must be used in place of $H$. Typical surcharge pressures exerted on sheet pile walls are caused by railroads, highways, dock loading facilities and merchandise, ore piles, and cranes.

4. **Unbalanced hydrostatic and seepage forces**: The previous discussion has assumed that the water levels on both sides of the sheet pile wall are at the same elevation. Depending on factors such as the watertightness of the sheet pile wall and the backfill permeability, it is possible that the groundwater level could be higher than the water level in front of the wall, in which case the wall would be subjected to water pressures. This condition could develop when there is a receding tide or a heavy rainstorm that causes a high groundwater table. A flow net can be used to determine the unbalanced hydrostatic and upward seepage forces in the soil in front of the sheet pile wall.

5. **Other loading conditions**: The sheet pile wall may have to be designed to resist the lateral loads due to ice thrust, wave forces, ship impact, mooring pull, and earthquake forces. If granular soil behind or in front of the sheet pile wall is in a loose state, it could be susceptible to liquefaction during an earthquake.

**Earthquake Analysis.** In the case of liquefaction of soil, the earthquake design pressures must be modified. As indicated in Sec. 10.3.2, higher pressures will be exerted on the back face of the wall if this soil should liquefy. Likewise, there will be less passive resistance if the soil in front of the sheet pile wall will liquefy during the design earthquake. Section 10.3.2 should be used as a guide in the selection of the pressures exerted on the sheet pile wall during the earthquake. Once these earthquake-induced pressures behind and in front of the wall are known, then the factor of safety for toe failure and the anchor pull force can be calculated in the same manner as outlined in the previous section.

**Example Problems.** Using the sheet pile wall diagram shown in Fig. 10.9, assume that the soil behind and in front of the sheet wall is uniform sand with a friction angle $\phi' = 33^\circ$, buoyant unit weight $\gamma_b = 64$ lb/ft$^3$, and above the groundwater table, the total unit weight
\( \gamma = 120 \text{ lb/ft}^3 \). Also assume that the sheet pile wall has \( H = 30 \text{ ft} \) and \( D = 20 \text{ ft} \), the water level in front of the wall is at the same elevation as the groundwater table which is located 5 ft below the ground surface, and the tieback anchor is located 4 ft below the ground surface. In the analysis, neglect wall friction.

**Static Design.** Calculate the factor of safety for toe kick-out and the tieback anchor force.

Equation (10.2):

\[
k_a = \tan^2 (45^\circ - \frac{\phi}{2}) = \tan^2 [45^\circ - \frac{\phi}{2}(33^\circ)] = 0.295
\]

Equation (10.4):

\[
k_p = \tan^2 (45^\circ + \frac{\phi}{2}) = \tan^2 [45^\circ + \frac{\phi}{2}(33^\circ)] = 3.39
\]

From 0 to 5 ft:

\[
P_a = \frac{1}{2} k_a \gamma (5)^2 = \frac{1}{2}(0.295)(120)(5)^2 = 400 \text{ lb/ft}
\]

From 5 to 50 ft:

\[
P_a = k_a \gamma (5) + \frac{1}{2} k_a \gamma D = 0.295(120)(5) + \frac{1}{2}(0.295)(64)(45)^2
\]

\[
P_a = 8000 + 19,100 = 27,100
\]

Equation (10.3) with \( \gamma_s \):

\[
P_p = \frac{1}{2} k_p \gamma D = \frac{1}{2}(3.39)(64)(20)^2 = 43,400 \text{ lb/ft}
\]

Moment due to passive force \( = 43,400(26 + \frac{\gamma_s}{20}) = 1.71 \times 10^6 \)

Neglecting \( P_{p,s} \).

Moment due to active force (at tieback anchor)

\[
= 8000 \left( 1 + \frac{45}{2} \right) + 19,100\left[ 1 + \frac{\gamma_s}{45} \right] = 7.8 \times 10^5
\]

\[
FS = \frac{\text{resisting moment}}{\text{destabilizing moment}} = \frac{1.71 \times 10^6}{7.8 \times 10^5} = 2.19
\]

\[
A_p = P_a - \frac{P_p}{FS} = 27,500 - \frac{43,400}{2.19} = 7680 \text{ lb/ft}
\]

For a 10-ft spacing, therefore,

\[
A_p = 10(7680) = 76,800 \text{ lb} = 76.8 \text{ kips}
\]

**Earthquake Analysis, Pseudostatic Method.** For the first earthquake analysis, assume that the sand behind, beneath, and in front of the wall has a factor of safety against liquefaction that is greater than 2.0. The design earthquake condition is \( a_{max} = 0.20g \). Using the pseudostatic approach [i.e., Eq. (10.7)], calculate the factor of safety for toe kick-out and the tieback anchor force.
Since the effect of the water pressure tends to cancel on both sides of the wall, use Eq. (10.7) and estimate $P_E$ based on the buoyant unit weight $\gamma_b = 64 \text{ lb/ft}^3$, or

$$P_E = \frac{1}{2} k_k \left( \frac{a_{\text{max}}}{g} \right) (H^2 \gamma_b) = \frac{1}{2} (0.295) \frac{1}{2} (0.20)(50)^2 (64) = 8690 \text{ lb/ft}$$

And $P_E$ acts at a distance of $\frac{1}{3}(H + D)$ above the bottom of the sheet pile wall.

**Moment due to** $P_E$ 

$$\text{Moment due to } P_E = 8690 \left[ \frac{1}{3} (50) \right] = 1.10 \times 10^5$$

**Total destabilizing moment**

$$7.80 \times 10^5 + 1.10 \times 10^5 = 8.90 \times 10^5$$

**Moment due to passive force**

$$\text{Moment due to passive force} = 1.71 \times 10^6$$

$$\text{FS} = \frac{\text{resisting moment}}{\text{destabilizing moment}} = \frac{1.71 \times 10^6}{8.90 \times 10^5} = 1.92$$

$$A_p = P_E + P_E - \frac{P_E}{\text{FS}} = 27,500 + 8690 - \frac{43,400}{1.92} = 13,600 \text{ lb/ft}$$

For a 10-ft spacing, therefore

$$A_p = 10(13,600) = 136,000 \text{ lb} = 136 \text{ kips}$$

**Earthquake Analysis, Liquefaction of Passive Wedge.** For the second earthquake analysis, assume that the sand located behind the retaining wall has a factor of safety against liquefaction greater than 2.0. Also assume that the upper 10 ft of sand located in front of the retaining wall will liquefy during the design earthquake, while the sand located below a depth of 10 ft has a factor of safety greater than 2.0. Calculate the factor of safety for toe kick-out and the tieback anchor force.

For the passive wedge:

- **0 to 10 ft:** Passive resistance = 0
- **At 10-ft depth:** Passive resistance = $k_p \gamma_d = 3.39(64)(10) = 2170 \text{ lb/ft}^2$
- **At 20-ft depth:** Passive resistance = $k_p \gamma_d = 3.39(64)(20) = 4340 \text{ lb/ft}^2$

$$\text{Passive force} = \frac{(2170 + 4340)}{2} (10) = 32,600 \text{ lb/ft}$$

$$\text{Moment due to passive force} = 2170(10)(45 - 4) + \frac{4340 - 2170}{2} (10)[40 + \frac{1}{3}(10) - 4]$$

$$= 890,000 + 463,000 = 1.35 \times 10^6$$

Including a pseudostatic force in the analysis gives these results:

$$P_E = \frac{1}{2} k_k \left( \frac{a_{\text{max}}}{g} \right) (H^2 \gamma_b) = \frac{1}{2} (0.295) \frac{1}{2} (0.20)(50)^2 (64) = 8690 \text{ lb/ft}$$

And $P_E$ acts at a distance of $\frac{1}{3}(H + D)$ above the bottom of the sheet pile wall.

**Moment due to** $P_E$ 

$$\text{Moment due to } P_E = 8690 \left[ \frac{1}{3} (50) \right] = 1.10 \times 10^5$$

**Total destabilizing moment**

$$7.80 \times 10^5 + 1.10 \times 10^5 = 8.90 \times 10^5$$
Moment due to passive force = $1.35 \times 10^6$

$$FS = \frac{\text{resisting moment}}{\text{destabilizing moment}} = \frac{1.35 \times 10^6}{8.90 \times 10^5} = 1.52$$

$$A_p = P_r + P_k - \frac{P_r}{FS} = 27,500 + 8690 - \frac{32,600}{1.52} = 14,700 \text{ lb/ft}$$

For a 10-ft spacing, therefore,

$$A_p = 10 \times (14,700) = 147,000 \text{ lb} = 147 \text{ kips}$$

**Earthquake Analysis, Liquefaction of Active Wedge.** For the third earthquake analysis, assume that the sand located in front of the retaining wall has a factor of safety against liquefaction greater than 2.0. However, assume that the submerged sand located behind the retaining wall will liquefy during the earthquake. Further assume that the tieback anchor will be unaffected by the liquefaction. Calculate the factor of safety for toe kick-out.

As indicated in Sec. 10.3.2, when the water levels are approximately the same on both sides of the retaining wall, use Eq. (10.1) with $k_A = 1$ [i.e., for $\phi' = 0, k_A = 1$, see Eq. (10.2)] and use $\gamma_b$ (buoyant unit weight) in place of $\gamma'$. As an approximation, assume that the entire 50 ft of soil behind the sheet pile wall will liquefy during the earthquake. Using Eq. (10.1), with $k_A = 1$ and $\gamma_b = 64 \text{ lb/ft}^3$,

$$P_L = \frac{1}{2}k_A \gamma_b(H + D)^2 = \frac{1}{2}(1.0)(64)(50)^2 = 80,000 \text{ lb/ft}$$

Moment due to liquefied soil = $80,000[\frac{1}{2}(50) - 4] = 2.35 \times 10^6$

Moment due to passive force = $1.71 \times 10^6$

$$FS = \frac{\text{resisting moment}}{\text{destabilizing moment}} = \frac{1.71 \times 10^6}{2.35 \times 10^5} = 0.73$$

### Summary of Values

<table>
<thead>
<tr>
<th>Example problem</th>
<th>Factor of safety for toe kick-out</th>
<th>$A_p$, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static analysis</td>
<td>2.19</td>
<td>76.8</td>
</tr>
<tr>
<td><strong>Earthquake</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudostatic method [Eq. (10.7)]</td>
<td>1.92</td>
<td>136</td>
</tr>
<tr>
<td>Partial passive wedge liquefaction*</td>
<td>1.52</td>
<td>147</td>
</tr>
<tr>
<td>Liquefaction of soil behind wall</td>
<td>0.73</td>
<td>—</td>
</tr>
</tbody>
</table>

*Pseudostatic force included for the active wedge.

As indicated by the values in this summary table, the sheet pile wall would not fail for partial liquefaction of the passive wedge. However, liquefaction of the soil behind the retaining wall would cause failure of the wall.
10.3.4 Summary

As discussed in the previous sections, the liquefaction of soil can affect the retaining wall in many different ways. It is also possible that even with a factor of safety against liquefaction greater than 1.0, there could be still be significant weakening of the soil, leading to a retaining wall failure. In summary, the type of analysis should be based on the factor of safety against liquefaction $F_{SL}$ as follows:

1. $F_{SL} \leq 1.0$: In this case, the soil is expected to liquefy during the design earthquake, and thus the design pressures acting on the retaining wall must be adjusted (see Sec. 10.3.2).

2. $F_{SL} > 2.0$: If the factor of safety against liquefaction is greater than about 2.0, the pore water pressures generated by the earthquake-induced contraction of the soil are usually small enough that they can be neglected. In this case, it could be assumed that the earthquake does not weaken the soil, and the pseudostatic analyses outlined in Sec. 10.2 could be performed.

3. $1.0 < F_{SL} \leq 2.0$: For this case, the soil is not anticipated to liquefy during the earthquake. However, as the loose granular soil contracts during the earthquake, there could still be a substantial increase in pore water pressure and hence weakening of the soil. Figure 5.15 can be used to estimate the pore water pressure ratio $r_u$ for various values of the factor of safety against liquefaction $F_{SL}$. The analysis would vary depending on the location of the increase in pore water pressure as follows:

   - **Passive wedge**: If the soil in the passive wedge has a factor of safety against liquefaction greater than 1.0 but less than 2.0, then the increase in pore water pressure would decrease the effective shear strength and the passive resisting force would be reduced [i.e., passive resistance $= P_p(1 - r_u)$].

   - **Bearing soil**: For an increase in the pore water pressure in the bearing soil, use the analysis in Sec. 8.3.

   - **Active wedge**: In addition to the pseudostatic force $P_e$ and the active earth pressure resultant force $P_a$, include a force that is equivalent to the anticipated earthquake-induced pore water pressure.

10.4 RETAINING WALL ANALYSES FOR WEAKENED SOIL

Besides the liquefaction of soil, many other types of soil can be weakened during the earthquake. In general, there are three cases:

1. **Weakening of backfill soil**: In this case, only the backfill soil is weakened during the earthquake. An example would be backfill soil that is susceptible to strain softening during the earthquake. As the backfill soil weakens during the earthquake, the force exerted on the back face of the wall increases. One design approach would be to estimate the shear strength corresponding to the weakened condition of the backfill soil and then use this strength to calculate the force exerted on the wall. The bearing pressure, factor of safety for sliding, factor of safety for overturning, and location of the resultant vertical force could then be calculated for this weakened backfill soil condition.

2. **Reduction in the soil resistance**: In this case, the soil beneath the bottom of the wall or the soil in the passive wedge is weakened during the earthquake. For example, the bearing soil could be susceptible to strain softening during the earthquake. As the bearing soil weakens during the earthquake, the wall foundation could experience additional settlement, a bearing capacity failure, sliding failure, or overturning failure. In addition, the weakening of
the ground beneath or in front of the wall could result in a shear failure beneath the retaining wall. One design approach would be to reduce the shear strength of the bearing soil or passive wedge soil to account for its weakened state during the earthquake. The settlement, bearing capacity, factor of safety for sliding, factor of safety for overturning, and factor of safety for a shear failure beneath the bottom of the wall would then be calculated for this weakened soil condition.

3. Weakening of the backfill soil and reduction in the soil resistance: This is the most complicated case and would require combined analyses of both items 1 and 2 as outlined above.

### 10.5 RESTRAINED RETAINING WALLS

#### 10.5.1 Introduction

As mentioned in Sec. 10.1.1, in order for the active wedge to be developed, there must be sufficient movement of the retaining wall. In many cases movement of the retaining wall is restricted. Examples include massive bridge abutments, rigid basement walls, and retaining walls that are anchored in nonyielding rock. These cases are often described as restrained retaining walls.

#### 10.5.2 Method of Analysis

To determine the static earth pressure acting on a restrained retaining wall, Eq. (10.1) can be utilized where the coefficient of earth pressure at rest \( k_0 \) is substituted for \( k_A \). For static design conditions of restrained retaining walls that have granular backfill, a commonly used value of \( k_0 \) is 0.5. Restrained retaining walls are especially susceptible to higher earth pressures induced by heavy compaction equipment, and extra care must be taken during the compaction of backfill for restrained retaining walls.

For earthquake conditions, restrained retaining walls will usually be subjected to larger forces compared to those retaining walls that have the ability to develop the active wedge. One approach is to use the pseudostatic method to calculate the earthquake force, with an increase to compensate for the unyielding wall conditions, or

\[
P_{ER} = \frac{P_E k_0}{k_A} \tag{10.22}
\]

where \( P_{ER} \) = pseudostatic force acting upon a restrained retaining wall, lb or kN
\( P_E \) = pseudostatic force assuming wall has the ability to develop the active wedge, i.e., use Eq. (10.7), (10.8), or (10.9), lb or kN
\( k_0 \) = coefficient of earth pressure at rest
\( k_A \) = active earth pressure coefficient, calculated from Eq. (10.2) or using the \( k_A \) equation in Fig. 10.3

#### 10.5.3 Example Problem

Use the example problem from Sec. 10.2.4 (i.e., Fig. 10.5), but assume that it is an unyielding bridge abutment. Determine the static and earthquake resultant forces acting on the restrained retaining wall. Neglect friction between the wall and backfill (\( \delta = \phi_w = 0 \)).
**Static Analysis.** Using a value of $k_0 = 0.5$ and substituting $k_0$ for $k_A$ in Eq. (10.1), we see the static earth pressure resultant force exerted on the restrained retaining wall is

$$P_R = \frac{1}{2}k_0 \gamma H^2 = \frac{1}{2}(0.5)(110)(20)^2 = 11000 \text{ lb per linear foot of wall}$$

The location of this static force is at a distance of $\frac{1}{3}H = 6.7 \text{ ft above the base of the wall.}$

**Earthquake Analysis.** Using the method outlined in Sec. 10.2.1, we find the value of $k_A = 0.333$ and $P_E = 2540 \text{ lb per linear foot of wall length}$ (see Sec. 10.2.4). Therefore, using Eq. (10.22), we have

$$P_{ER} = P_E \frac{k_A}{k_0} = 2540 \frac{0.333}{0.5} = 3800 \text{ lb per linear foot of wall}$$

The location of this pseudostatic force is assumed to act at a distance of $\frac{1}{3}H = 13.3 \text{ ft above the base of the wall.}$

In summary, the resultant earth pressure forces acting on the retaining wall are static $P_R = 11000 \text{ lb/ft acting at a distance of 6.7 ft above the base of the wall and earthquake } P_{ER} = 3800 \text{ lb/ft acting at a distance of 13.3 ft above the base of the wall.}$

### 10.6 TEMPORARY RETAINING WALLS

#### 10.6.1 Static Design

Temporary retaining walls are often used during construction, such as for the support of the sides of an excavation that is made below grade to construct the building foundation. If the temporary retaining wall has the ability to develop the active wedge, then the basic active earth pressure principles described in Sec. 10.1.1 can be used for the design of the temporary retaining walls.

Especially in urban areas, movement of the temporary retaining wall may have to be restricted to prevent damage to adjacent property. If movement of the retaining wall is restricted, the earth pressures will typically be between the active ($k_A$) and at-rest ($k_0$) values.

For some projects, temporary retaining walls may be constructed of sheeting (such as sheet piles) that are supported by horizontal braces, also known as struts. Near or at the top of the temporary retaining wall, the struts restrict movement of the retaining wall and prevent the development of the active wedge. Because of this inability of the retaining wall to deform at the top, earth pressures near the top of the wall are in excess of the active ($k_A$) pressures. At the bottom of the wall, the soil is usually able to deform into the excavation, which results in a reduction in earth pressure. Thus the earth pressures at the bottom of the excavation tend to be constant or even decrease, as shown in Fig. 10.10.

The earth pressure distributions shown in Fig. 10.10 were developed from actual measurements of the forces in struts during the construction of braced excavations. In Fig. 10.10, case $a$ shows the earth pressure distribution for braced excavations in sand and cases $b$ and $c$ show the earth pressure distribution for clays. In Fig. 10.10, the distance $H$ represents the depth of the excavation (i.e., the height of the exposed wall surface). The earth pressure distribution is applied over the exposed height $H$ of the wall surface with the earth pressures transferred from the wall sheeting to the struts (the struts are labeled with forces $F_1, F_2$, etc.).

Any surcharge pressures, such as surcharge pressures on the ground surface adjacent to the excavation, must be added to the pressure distributions shown in Fig. 10.10. In addition, if the sand deposit has a groundwater table that is above the level of the bottom of the excavation, then water pressures must be added to the case $a$ pressure distribution shown in Fig. 10.10.
10.34

CHAPTER TEN

Figure 10.10 Earth pressure distribution on temporary braced walls. (From NAVFAC DM-7.2 1982, originally developed by Terzaghi and Peck 1967.)

(a) SAND

\[ \sigma_h = 0.65 K_A \gamma H \]

WHERE \( K_A = \tan^2 (45 - \phi/2) \)

(b) SOFT TO MEDIUM CLAY

\( (N_0 > 6) \)

For clays base the selection on

\[ N_0 = \gamma H/c \]

\[ \sigma_h = K_A \gamma H \]

\[ K_A = 1 - m \frac{2 \gamma H}{c} \]

For clays where cut is underlain by deep soft

\[ m = 1 \] except where cut is normally consolidated

clay, then \( m = 0.4 \)

Assume hinges at strut locations for calculating

strut forces

(c) STIFF CLAY

\( (N_0 < 4) \)

For \( 4 < N_0 < 6 \), use larger of diagrams (b) and (c).

\[ \sigma_h > 0.2 \gamma H; \sigma_h = 0.4 \gamma H \]

Use lower value when movements are minimal and short

construction period.
Because the excavations are temporary (i.e., short-term condition), the undrained shear strength \( (s_u) \) is used for the analysis of the earth pressure distributions for clay. The earth pressure distributions for clay (i.e., cases \( b \) and \( c \)) are not valid for permanent walls or for walls where the groundwater table is above the bottom of the excavation.

### 10.6.2 Earthquake Analysis

Since temporary retaining walls are usually only in service for a short time, the possibility of earthquake effects is typically ignored. However, in active seismic zones or if the consequence of failure could be catastrophic, it may be prudent to perform an earthquake analysis. Depending on whether the wall is considered to be yielding or restrained, the analysis would be based on the data in Sec. 10.2 or Sec. 10.5. Weakening of the soil during the design earthquake and its effects on the temporary retaining wall should also be included in the analysis.

### 10.7 PROBLEMS

The problems have been divided into basic categories as indicated below.

**Pseudostatic Method**

10.1 Using the retaining wall shown in Fig. 10.4, assume \( H = 4 \) m, the thickness of the reinforced concrete wall stem = 0.4 m, the reinforced concrete wall footing is 3 m wide by 0.5 m thick, the ground surface in front of the wall is level with the top of the wall footing, and the unit weight of concrete = 23.5 kN/m\(^3\). The wall backfill will consist of sand having \( \phi = 32^\circ \) and \( \gamma = 20 \) kN/m\(^3\). Also assume that there is sand in front of the footing with these same soil properties. The friction angle between the bottom of the footing and the bearing soil \( \delta = 38^\circ \). For the condition of a level backfill and neglecting the wall friction on the backside of the wall and the front side of the footing, determine the resultant normal force \( N \) and the distance of \( N \) from the toe of the footing, the maximum bearing pressure \( q' \) and the minimum bearing pressure \( q'' \) exerted by the retaining wall foundation, factor of safety for sliding, and factor of safety for overturning for static conditions and earthquake conditions [using Eq. (10.7)] if \( a_{\text{max}} = 0.20g \). Answer: Static conditions: \( N = 68.2 \) kN/m and location = 1.16 m from toe, \( q' = 37.9 \) kPa and \( q'' = 7.5 \) kPa, FS for sliding = 1.17, and FS for overturning = 2.2. Earthquake conditions: \( P_{\text{AE}} = 17.7 \) kN/m, \( N \) is not within the middle third of the footing, FS for sliding = 0.86, FS for overturning = 1.29.

10.2 Solve Prob. 10.1, using Eq. (10.8). Answer: Static values are the same. Earthquake conditions: \( P_{\text{AE}} = 24 \) kN/m, \( N \) is not within the middle third of the footing, FS for sliding = 0.78, FS for overturning = 1.18.

10.3 Solve Prob. 10.1, but include wall friction in the analysis (use Coulomb’s earth pressure equation, Fig. 10.3). Assume the friction angle between the backside of the retaining wall and the backfill is equal to \( \frac{1}{4} \) of \( \phi \) (that is, \( \phi_w = \frac{1}{4} \phi = 24^\circ \)). Use Eq. (10.9) for the earthquake analysis. Answer: Static condition: \( N = 86.1 \) kN/m and location = 1.69 m from toe, \( q' = 39.6 \) kPa and \( q'' = 17.8 \) kPa, FS for sliding = 1.78, and FS for overturning = \( \infty \). Earthquake conditions: \( P_{\text{AE}} = 68.5 \) kN/m, \( N \) is 1.51 m from the toe of the footing, \( q' = q'' = 32.0 \) kPa, FS for sliding = 1.26, FS for overturning = \( \infty \).

10.4 Using the retaining wall shown at the top of Fig. 10.2b (i.e., a cantilevered retaining wall), assume \( H = 4 \) m, the thickness of the reinforced concrete wall stem = 0.4 m and the
wall stem is located at the centerline of the footing, the reinforced concrete wall footing is 2 m wide by 0.5 m thick, the ground surface in front of the wall is level with the top of the wall footing, and the unit weight of concrete = 23.5 kN/m$^3$. The wall backfill will consist of sand having $\phi = 32^\circ$ and $\gamma_s = 20$ kN/m$^3$. Also assume that there is sand in front of the footing with these same soil properties. The friction angle between the bottom of the footing and the bearing soil $\delta = 24^\circ$. For the condition of a level backfill and assuming total mobilization of the shear strength along the vertical plane at the heel of the wall, calculate the resultant normal force $N$ and the distance of $N$ from the toe of the footing, the maximum bearing pressure $q'$ and the minimum bearing pressure $q''$ exerted by the retaining wall foundation, factor of safety for sliding, and factor of safety for overturning for static conditions and earthquake conditions [using Eq. (10.9)] if $a_{max} = 0.20g$. Answer: Static conditions: $N = 136$ kN/m and location = 1.05 m from toe, $q' = 78.1$ kPa and $q'' = 57.8$ kPa, FS for sliding = 1.72, and FS for overturning = 2.73. Earthquake conditions: $P_{AE} = 71.2$ kN/m, $N = 0.94$ m from the toe of the footing, $q' = 88.6$ kPa and $q'' = 61.5$ kPa, FS for sliding = 1.17, FS for overturning = 2.9.

10.5 For the example problem shown in Fig. 10.5, assume that there is a vertical surcharge pressure of 200 lb/ft$^2$ located at ground surface behind the retaining wall. Calculate the factor of safety for sliding and the factor of safety for overturning, and determine if $N$ is within the middle third of the retaining wall foundation for the static conditions and earthquake conditions [using Eq. (10.9)] if $a_{max} = 0.20g$. Answer: Static conditions: FS for sliding = 1.48, FS for overturning = 2.64, and $N$ is not within the middle third of the retaining wall foundation. Earthquake conditions: FS for sliding = 1.02, FS for overturning = 0.91, and $N$ is not within the middle third of the retaining wall foundation.

10.6 For the example problem shown in Fig. 10.5, assume that the ground surface behind the retaining wall slopes upward at a 3:1 (horizontal:vertical) slope inclination. Calculate the factor of safety for sliding and factor of safety for overturning, and determine if $N$ is within the middle third of the retaining wall foundation for the static conditions and earthquake conditions [using Eq. (10.9)] if $a_{max} = 0.20g$. Answer: Static conditions: FS for sliding = 1.32, FS for overturning = 2.73, and $N$ is not within the middle third of the retaining wall foundation. Earthquake conditions: FS for sliding = 0.72, FS for overturning = 1.06, and $N$ is not within the middle third of the retaining wall foundation.

10.7 Use the data from the example problem in Sec. 10.2.5, and assume that there is a vertical surcharge pressure of 200 lb/ft$^2$ located at ground surface behind the mechanically stabilized earth retaining wall. Calculate the factor of safety for sliding, factor of safety for overturning, and maximum pressure exerted by the base of the mechanically stabilized earth retaining wall for static and earthquake conditions. Answer: Static conditions: FS for sliding = 1.73, FS for overturning = 3.78, and maximum pressure $q' = 4300$ lb/ft$^2$. Earthquake conditions: FS for sliding = 1.29, FS for overturning = 2.3, and $N$ is not within the middle third of the base of the wall.

10.8 Use the data from the example problem in Sec. 10.2.5, and assume that the ground surface behind the mechanically stabilized earth retaining wall slopes upward at a 3:1 (horizontal:vertical) slope inclination. Also assume that the 3:1 slope does not start at the upper front corner of the rectangular reinforced soil mass (such as shown in Fig. 10.8), but instead the 3:1 slope starts at the upper back corner of the rectangular reinforced soil mass. Calculate the factor of safety for sliding, factor of safety for overturning, and maximum pressure exerted by the retaining wall foundation for the static and earthquake conditions, using the equations in Fig. 10.3. Answer: Static conditions: FS for sliding = 1.60, FS for overturning = 3.76, and maximum pressure $q' = 4310$ lb/ft$^2$. Earthquake conditions: FS for sliding = 0.81, FS for overturning = 1.91, and $N$ is not within the middle third of the base of the wall.

10.9 For the example problem in Sec. 10.2.5, the internal stability of the mechanically stabilized zone is to be checked by using wedge analysis. Assume a planer slip surface that
is inclined at an angle of 61° (that is, \( \alpha = 61° \)) and passes through the toe of the mechanically stabilized zone. Also assume that the mechanically stabilized zone contains 40 horizontal layers of Tensar SS2 geogrid which has an allowable tensile strength = 300 lb/ft of wall length for each geogrid. In the wedge analysis, these 40 layers of geogrid can be represented as an allowable horizontal resistance force = 12,000 lb/ft of wall length (that is, 40 layers times 300 lb). If the friction angle \( \phi \) of the sand = 32° in the mechanically stabilized zone, calculate the factor of safety for internal stability of the mechanically stabilized zone, using the wedge analysis for static and earthquake conditions. Answer: Static conditions: \( F = 1.82 \); earthquake conditions: \( FS = 1.29 \).

**Sheet Pile Wall Analyses for Liquefied Soil**

10.10 For the example problem in Sec. 10.3.3, assume that there is a uniform vertical surcharge pressure = 200 lb/ft² applied to the ground surface behind the sheet pile wall. Calculate the factor of safety for toe kick-out and the anchor pull force for the static condition and the earthquake conditions, using the pseudostatic method, and for partial liquefaction of the passive wedge. Answer: See App. E for solution.

10.11 For the example problem in Sec. 10.3.3, assume that the ground surface slopes upward at a 3:1 (horizontal:vertical) slope ratio behind the sheet pile wall. Calculate the factor of safety for toe kick-out and the anchor pull force for the static condition and the earthquake conditions, using the pseudostatic method, and for partial liquefaction of the passive wedge. Answer: See App. E for solution.

10.12 For the example problem in Sec. 10.3.3, assume that the ground in front of the sheet pile wall (i.e., the passive earth zone) slopes downward at a 3:1 (horizontal:vertical) slope ratio. Calculate the factor of safety for toe kick-out for the static condition and the earthquake conditions, using the pseudostatic method. Answer: Static condition: \( FS \) for toe kick-out = 1.18; earthquake condition: \( FS \) for toe kick-out = 1.04.

10.13 For the example problem in Sec. 10.3.3, assume that the anchor block is far enough back from the face of the sheet pile wall that it is not in the active zone. Also assume that the anchor block is located at a depth of 3 to 5 ft below ground surface, it is 5 ft by 5 ft in plan dimensions, and it consists of concrete that has a unit weight of 150 lb/ft³. Further assume that the tieback rod is located at the center of gravity of the anchor block. For friction on the top and bottom of the anchor block, use a friction coefficient = \( \mu \phi \), where \( \phi \) = friction angle of the sand. Determine the lateral resistance of the anchor block for static conditions and for earthquake conditions, assuming that all the soil behind the retaining wall will liquefy during the earthquake. Answer: Static condition: lateral resistance = 26.6 kips; earthquake conditions: lateral resistance = 0.

**Braced Excavations**

10.14 A braced excavation will be used to support the vertical sides of a 20-ft-deep excavation (that is, \( H = 20 \) ft in Fig. 10.10). If the site consists of a sand with a friction angle \( \phi = 32° \) and a total unit weight \( \gamma = 120 \) lb/ft², calculate the earth pressure \( \sigma_c \) and the resultant earth pressure force acting on the braced excavation for the static condition and the earthquake condition (using Eq. (10.7)) if \( a_{max} = 0.20g \). Assume the groundwater table is well below the bottom of the excavation. Answer: Static condition: \( \sigma_c = 480 \) lb/ft² and the resultant force = 9600 lb per linear foot of wall length. Earthquake condition: \( P_e = 2700 \) lb per linear foot of wall length.

10.15 Solve Prob. 10.14, but assume the site consists of a soft clay having an undrained shear strength \( s_u = 300 \) lb/ft² (that is, \( c = s_u = 300 \) lb/ft²) and use Eq. (10.8).
Answer: Static condition: \( \sigma_s = 1200 \text{ lb/ft}^2 \), and the resultant force = \( 21,000 \text{ lb per linear foot of wall length} \). Earthquake condition: \( P_{h} = 3600 \text{ lb per linear foot of wall length} \).

10.16 Solve Prob. 10.15, but assume the site consists of a stiff clay having an undrained shear strength \( s_u = 1200 \text{ lb/ft}^2 \) and use the higher earth pressure condition (that is, \( \sigma_{s2} \)). Answer: Static condition: \( \sigma_{s2} = 960 \text{ lb/ft}^2 \), and resultant force = \( 14,400 \text{ lb per linear foot of wall length} \). Earthquake condition: \( P_{h} = 3600 \text{ lb per linear foot of wall length} \).

Subsoil Profiles

10.17 Use the data from Prob. 6.15 and Fig. 6.15 (i.e., sewage site at Niigata). Assume the subsoil profile represents conditions behind a retaining wall. Also assume that the type of retaining wall installed at the site is a concrete box structure, having height = 8 m, width = 5 m, and total weight of the concrete box structure = 823 kilonewtons per linear meter of wall length. The soil behind the retaining wall is flush with the top of the concrete box structure. The water level in front of the retaining wall is at the same elevation as the groundwater table behind the wall. The effective friction angle \( \phi' \) of the soil can be assumed to be equal to 30°, wall friction along the back face of the wall can be neglected, and the coefficient of friction along the bottom of the wall = \( 2/3 \phi' \). In addition, the ground in front of the wall is located 1 m above the bottom of the wall, and the subsoil profile in Fig. 6.15 starting at a depth of 7 m can be assumed to be applicable for the soil in front of the wall. For the static conditions and earthquake conditions, determine the resultant normal force \( N \) and the distance of \( N \) from the toe of the wall, the maximum bearing pressure \( q' \) and the minimum bearing pressure \( q'' \) exerted by the retaining wall foundation, factor of safety for sliding, and factor of safety for overturning. Answer: Static conditions: \( N = 450 \text{ kN/m} \) and location = 1.89 m from toe, \( q' = 156 \text{ kPa} \) and \( q'' = 24 \text{ kPa} \), FS for sliding = 1.66, and FS for overturning = 4.1. Earthquake conditions: \( N = 450 \text{ kN/m} \), \( N \) is not within the middle third of the footings, FS for sliding = 0.55, FS for overturning = 1.36.

Submerged Backfill Condition

10.18 A cantilevered retaining wall (3 m in height) has a granular backfill with \( \phi = 30^\circ \) and \( \gamma_t = 20 \text{ kN/m}^3 \). Neglect wall friction, and assume the drainage system fails and the water level rises 3 m above the bottom of the retaining wall (i.e., the water table rises to the top of the retaining wall). Determine the initial active earth pressure resultant force \( P_a \) and the resultant force (due to earth plus water pressure) on the wall due to the rise in water level. For the failed drainage system condition, also calculate the total force on the wall if the soil behind the retaining wall should liquefy during the earthquake. For both the static and earthquake conditions, assume that there is no water in front of the retaining wall (i.e., only a groundwater table behind the retaining wall). Answer: Static condition: \( P_a = 30 \text{ kN/m} \) (initial condition). With a rise in water level the force acting on the wall = 59.4 kN/m. Earthquake condition: \( P_a = 90 \text{ kN/m} \).